Chapter 0
Organization and Overview

Convex Optimization for Machine Learning & Computer Vision
SS 2018

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Department of Informatics
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Organization

Whether this lecture fits you?

Prerequisites

- Background in mathematical analysis and linear algebra
- Numerical implementation in Matlab or Python
- Interest in mathematical theory (know why!)

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Nice plus (but not necessary)

- Experience in machine learning or computer vision
  e.g., CV I & II, ML for CV, Probab. Graphical Models in CV.
- Knowledge in continuous optimization
  e.g., Nonlinear Optimization.
- Knowledge in functional analysis
  Further understanding of the theory in full generality.

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Course overview

Lectures

1. Theory of convex analysis
2. Design and analysis of optimization algorithms
3. Selected topic: Stochastic optimization

Applications are mostly covered in exercise session...

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Exercise session

Organizers: Emanuel Laude and Zhenzhang Ye

- Exercise sheets covering the content of the lecture will be passed out every Wednesday.
- Exercises contain theoretical as well as programming questions.
- If solutions have obviously been copied, both groups will get 0 points.
- You may work on the exercises in groups of two.
- You are encouraged to present your solution on board at exercise class.
- To get a 0.3 grade bonus, you need to complete 75% of the total exercise points.

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Contact us

Miscellaneous info

- Tao’s office: 02.09.061
- Emanuel’s office: 02.09.039
- Zhenzhang’s office: 02.09.037
- Office hours: Please write an email.
- Lecture: Starts at quarter past. Short break in between.
- Course website:
  `https://vision.in.tum.de/teaching/ss2018/cvx4cv`

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First Glimpse of the Course

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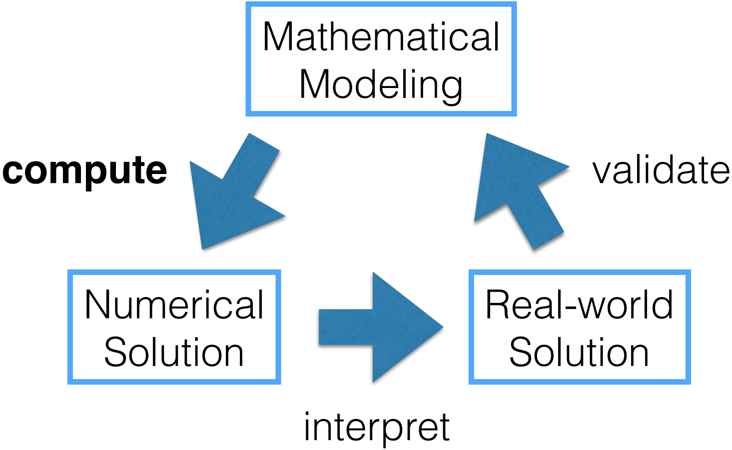
Driving cycle



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An example from computer vision / machine learning

- Image segmentation / multi-labeling:

image

segmentation ($L = 4$)

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An example from computer vision / machine learning

- Image segmentation / multi-labeling:

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segmentation ($L = 4$)

- Variational method for finding label function $u : \Omega \to \Delta^L$

$$\text{minimize} \sum_{j \in \Omega} \Big(\delta\{u_j \in \Delta^L\} + \langle u_j, f_j \rangle \Big) + \alpha \sum_{l=1}^{L} \sum_i \omega_i \cdot (\nabla u^l)_i,$$

where

- Pointwise constraint: Δ^L is the unit $(L-1)$-simplex.
- Unary term: $f : \Omega \to \mathbb{R}^L$ is a pre-computed vector.
- Pairwise term: $\sum_i \omega_i \cdot (\nabla u^l)_i$ is the weighted total-variation.

Prototypical workflow

- The variational model

$$\text{minimize} \sum_{j \in \Omega} \Big(\delta\{u_j \in \Delta^L\} + \langle u_j, f_j \rangle \Big) + \alpha \sum_{l=1}^{L} \sum_{i} \omega_i \cdot (\nabla u^l)_i,$$

  is a special case of **convex optimization**

$$\text{minimize } J(u) + \delta\{u \in C\},$$

  with **convex objective** J and **convex constraint** C.

- This course is about **theory** and **practice** for solving convex optimization (arising from computer vision and machine learning).

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Prototypical workflow

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$$\min_u F(Ku) + G(u),$$

where F, G are convex functions, K is a linear operator.

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$$\max_p \min_u \langle Ku, p \rangle - F^*(p) + G(u),$$

where F^* is the *convex conjugate* of F.

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$$\max_{p} \min_{u} \langle Ku, p \rangle - F^{*}(p) + G(u),$$

where F^{*} is the *convex conjugate* of F.

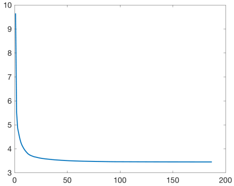
- Apply an iterative scheme (e.g. PDHG):

$$u^{k+1} = \arg\min_{u} \left\langle u, K^{\top} p^{k} \right\rangle + G(u) + \frac{s}{2}\|u - u^{k}\|^{2},$$

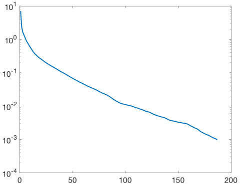
$$p^{k+1} = \arg\min_{p} - \left\langle K(2u^{k+1} - u^{k}), p \right\rangle + F^{*}(p) + \frac{t}{2}\|p - p^{k}\|^{2}.$$

Questions of our concern

energy value



primal-dual gap



- Does a minimizer always exist?
- How to characterize a minimizer?
- How to derive an optimization algorithm?
- How to analyze/guarantee the convergence?
- How to accelerate the convergence?
- Efficient implementation, etc.

Ready to start?