Convex Optimization for Machine Learning and Computer Vision

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Weekly Exercises 10

Room: 02.09.023
Wednesday, 04.07.2018, 12:15-14:00
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Consesus optimization (4+4 Points)

Exercise 1 (8 Points)**.** Consider the *consensus optimization* problem:

$$\begin{aligned} \min_{\{x_i\}_{i=1}^l \subset \mathbb{R}^n,\, x_0 \in \mathbb{R}^n} \quad & \sum_{i=1}^{l} f_i(x_i) \\ \text{subject to} \quad & x_i = x_0 \quad \forall i \in \{1, 2, ..., l\}. \end{aligned} \tag{1}$$

Here each function $f_i : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$ is proper, convex, and lower-semicontinuous.

- Write down the augmented Lagrangian functional for (1) (which will involve multipliers $\{y_i\}_{i=1}^l \subset \mathbb{R}^n$).
- Formulate an alternating direction of multipliers (ADMM) method for (1). Update the variables in the order of $\{x_i\}_{i=1}^l$, $\{y_i\}_{i=1}^l$, x_0.
- One can interpret the ADMM scheme in (b) as a generalized proximal iteration on $(x_0, \{y_i\}_{i=1}^l)$:

$$0 \in M \begin{bmatrix} x_0^{k+1} - x_0^k \\ y_1^{k+1} - y_1^k \\ \vdots \\ y_l^{k+1} - y_l^k \end{bmatrix} + R \left(\begin{bmatrix} x_0^{k+1} \\ y_1^{k+1} \\ \vdots \\ y_l^{k+1} \end{bmatrix} \right).$$

  Identify the positive semidefinite matrix M and the monotone operator R in the above equation.