Convex Optimization for Machine Learning and Computer Vision

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Weekly Exercises 9

Room: 02.09.023
Wednesday, 27.06.2018, 12:15-14:00
Submission deadline: Monday, 25.06.2018, 16:15, Room 02.09.023

Convergence Analysis (10+4 Points)

Exercise 1 (4 Points)**.** Show following properties of monotone operator:

- T is monotone, $\lambda \geq 0$. Then λT is monotone.
- R, S are monotone, $\lambda \geq 0$. Then $R + \lambda S$ is monotone.

Exercise 2 (6 Points)**.** Denote $\Pi_C(x)$ as the projection of point x onto a set C. Show following properties:

- Projection onto a convex set C is a monotone operator.
- Show that T is firmly nonexpansive, if and only if $||Tx - Ty||^2 \leq \langle x - y, Tx - Ty \rangle, \ \forall x, y$.
- Given a nonempty closed convex set C, show that Π_c is firmly nonexpansive. Hint: you might use that $\langle y - \Pi_C(x), x - \Pi_C(x) \rangle \leq 0, \forall y \in C$

Exercise 3 (4 Points)**.** Prove the theorem from the lecture:

Let C be a nonempty, closed, convex subset of $\mathbb{R}^n$. For each $i \in \{1, ..., m\}$, let $\alpha_i \in (0, 1)$, $\omega_i \in (0, 1)$ and $\Phi_i : C \to \mathbb{R}^n$ be an α_i-averaged operator. If $\sum_{i=1}^m \omega_i = 1$ and $\alpha = \max_{1 \leq i \leq m} \alpha_i$, then

$$\Phi = \sum_{i=1}^{m} \omega_i \Phi_i$$

is α-averaged.