TU München
Fakultät für Informatik
PD Dr. Rudolph Triebel
John Chiotellis, Maximilian Denninger

Machine Learning for Computer Vision

Topic: Linear Algebra

Note: *This exercise sheet is made to help you refresh some important concepts of Linear Algebra that are relevant for this course. It is not meant to be a homework assignment. Nevertheless being familiar and having these concepts fresh in mind will help you and save you time when studying the topics of the course.*

Exercise 1: Warm up

a) What multiple of $a = (1, 1, 1)$ is closest to the point $b = (2, 4, 4)$? Find also the closest point to a on the line through b.

b) Prove that the trace of $P = aa^T/a^Ta$ always equals 1.

c) Show that the length of Ax equals the length of A^Tx if $AA^T = A^TA$.

d) Which 2×2 matrix projects the x,y plane onto the line $x + y = 0$?

Exercise 2: Determinants

a) If a square matrix A has determinant $\frac{1}{2}$, find $\det(2A)$, $\det(-A)$, $\det(A^2)$ and $\det(A^{-1})$.

b) Find the determinants of

$$A = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} \quad , \quad U = \begin{bmatrix} 4 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \; U^T \text{ and } U^{-1}$$

Exercise 3: Eigenvalues and Eigenvectors

a) Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}, \text{ their traces and their determinants.}$$

b) Using the characteristic polynomial, find the relationship between the trace, the determinants and the eigenvalues of any square matrix A.

c) Diagonalize the unitary matrix V to reach $V = U\Lambda U^*$. All $|\lambda| = 1$. $V = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix}$

d) Suppose T is a 3×3 upper triangular matrix with entries t_{ij}. Compare the entries of T^*T and TT^*. Show that if they are equal, then T must be diagonal. (All normal triangular matrices are diagonal)

Exercise 4: Singular Value Decomposition

a) Find the singular values and singular vectors of

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$$

b) Explain how $U\Sigma V^T$ expresses A as a sum of r rank-1 matrices: $A = \sigma_1 u_1 v_1^T + \ldots + \sigma_r u_r v_r^T$

c) If A changes to $4A$ what is the change in the SVD?

What is the SVD for A^T and for A^{-1} ?

d) Find the SVD and the pseudoinverse of $A = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$