

## Machine Learning for Computer Vision

21. April 2018  
Topic: Regression

### Exercise 1: Basic model

Consider a linear regression model with basis functions  $\phi(x)$  as presented in the lecture. Suppose we have observed  $N$  data points  $\{x_i, t_i\}_{i=1\dots N}$ .

- What do we need to estimate if we want to fit this model?
- What would be the optimal solution in the sense of sum-of-squares error?
- Can you define and solve the problem in a probabilistic way?

*Hint: You have to make some assumptions.*

### Exercise 2: Bayesian Update

Now, we assume a Gaussian prior distribution for the weights:

$$p(\mathbf{w}) = \mathcal{N}(\mathbf{w}|m_0, S_0)$$

The posterior distribution after the  $N$  observed points is

$$p(\mathbf{w}|\mathbf{t}) = \mathcal{N}(\mathbf{w}|m_N, S_N)$$

with

$$m_N = S_N(S_0^{-1}m_0 + \sigma^{-2}\Phi^T\mathbf{t}) \quad \text{and} \quad S_N = S_0^{-1} + \sigma^{-2}\Phi^T\Phi.$$

We observe a new data point  $(x_{N+1}, t_{N+1})$ .

- What is the new posterior?
- What is the difference between this model and the model of the previous exercise?  
What are the names of the two methods?

### Exercise 3: Quadcopter (Programming)

We are testing a tracking program and we evaluate it with the help of a quadcopter. The quadcopter sends estimates of its velocity and the tracking program estimates its global position with respect to the quadcopter's initial position (before flying).

a) The tracker yields these tracked position estimates at a frequency of  $1Hz$ :

$$\mathcal{T} = \left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1.08 \\ 1.68 \\ 2.38 \end{pmatrix} \begin{pmatrix} -0.83 \\ 1.82 \\ 2.49 \end{pmatrix} \begin{pmatrix} -1.97 \\ 0.28 \\ 2.15 \end{pmatrix} \begin{pmatrix} -1.31 \\ -1.51 \\ 2.59 \end{pmatrix} \begin{pmatrix} 0.57 \\ -1.91 \\ 4.32 \end{pmatrix} \right\}$$

Plot the trajectory through these data points with your tool of choice.

- b) Assuming the quadcopter flies with constant speed, which speed does it have? What is the residual error of the estimation?
- c) Now assume that the quadcopter flies with constant acceleration. What is the residual error now? Is the error higher or lower? Why?
- d) According to our last model, what is the quadcopter's most likely position in the next second?

*Hint for b) and c): Use the Polynomial Regression method introduced in the lecture.*