Multiple View Geometry: Exercise Sheet 6<br>Prof. Dr. Daniel Cremers, Nikolaus Demmel, Marvin Eisenberger<br>Computer Vision Group, TU Munich<br>http://vision.in.tum.de/teaching/ss2018/mvg2018

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## Part I: Theory

The following exercises should be solved at home. Writing down your answer will help you present and discuss the answer during the tutorial.

1. The essential matrix $E=\hat{T} R$ has the singular value decomposition $E=U \Sigma V^{T}$. Let $R_{Z}\left( \pm \frac{\pi}{2}\right)$ be the rotation by $\pm \frac{\pi}{2}$ around the $z$-axis.

Show the following properties:
(a) $\hat{T} \in \operatorname{so}(3) \quad$ (i.e. $\hat{T}$ is a skew-symmetric matrix)
(b) $R \in \mathrm{SO}$ (3) (i.e. $R$ is a rotation matrix)

Hint: Use the equalities: $\hat{T}=U R_{Z}\left( \pm \frac{\pi}{2}\right) \Sigma U^{\top} \quad$ and $\quad R=U R_{Z}\left( \pm \frac{\pi}{2}\right)^{\top} V^{\top}$. (see Chapter 5, Slide 9)
2. Consider the matrices $E=\hat{T} R$ and $H=R+T u^{\top}$ with $R \in \mathbb{R}^{3 \times 3}$ and $T, u \in \mathbb{R}^{3}$. Show that the following holds:
(a) $E=\hat{T} H$
(b) $H^{\top} E+E^{\top} H=0$
3. Let $F \in \mathbb{R}^{3 \times 3}$ be the fundamental matrix for the cameras $C_{1}$ and $C_{2}$. Show that the following holds for the epipoles $e_{1}$ and $e_{2}$ :

$$
F e_{1}=0 \quad \text { and } \quad e_{2}^{\top} F=0
$$

## Part II: Practical Exercises

This exercise is to be solved during the tutorial.

1. Download the package ex6.zip from the website and extract the images batinria0.tif and batinrial.tif.
2. Get the 2D coordinates of corresponding point pairs: Show the first image and mark at least 8 points. You can retrieve the pixel coordinates of mouse clicks with the command $[\mathrm{x}, \mathrm{y}]$ $=$ ginput ( $g \subset f$ ). Then show the second image and click at the corresponding points in the same order. Again you can get the pixel coordinates with ginput. Now you should have the 2 D coordinates of corresponding point pairs.
3. Implement the 8 -point algorithm from the lecture and run it with these point pairs. To this end, you have to transform the coordinates. The intrinsic camera matrices are:

$$
\begin{aligned}
& K 1=\left(\begin{array}{ccc}
844.310547 & 0 & 243.413315 \\
0 & 1202.508301 & 281.529236 \\
0 & 0 & 1
\end{array}\right) \\
& K 2=\left(\begin{array}{ccc}
852.721008 & 0 & 252.021805 \\
0 & 1215.657349 & 288.587189 \\
0 & 0 & 1
\end{array}\right)
\end{aligned}
$$

4. Reconstruct the depths of the points as described in Chapter 5 on Slides 17 and 18.

Hints:

- The file additional_information.txt provides $K 1$ and $K 2$
- Use kron and reshape
- It might occur that one of the matrices $U$ or $V$ of the SVD of $E$ has a determinant less than zero. In this case, determine the SVD of $-E$.

