



Multiple View Geometry: Solution Sheet 7

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Part I: Theory

1. (a) l is coimage of L , and therefore l is normal vector to the plane that is determined by the camera position and L .

$$\begin{aligned} &\Rightarrow \begin{aligned} l^T x_1 &= 0 \\ l^T x_2 &= 0. \end{aligned} \\ &\Rightarrow l \sim x_1 \times x_2 = \hat{x}_1 x_2. \end{aligned}$$

l_1 and l_2 are normal vectors to the planes through camera position and L_1, L_2 respectively.

$$\begin{aligned} &\Rightarrow \begin{aligned} l_1^T x &= 0 \\ l_2^T x &= 0 \end{aligned} \\ &\Rightarrow x \sim l_1 \times l_2 = \hat{l}_1 \hat{l}_2. \end{aligned}$$

- (b) i. $l_1 \sim \hat{x}u$:
 x is in the preimage of L_1 . $\Rightarrow l_1^T x = 0$.
 \exists point $u \neq p$ in L_1 . $\Rightarrow l_1^T u = 0$
 $\Rightarrow l_1 \sim \hat{x}u$.
- ii. $l_2 \sim \hat{x}v$: analog to i.
- iii. $x_1 \sim \hat{l}r$:
 x_1 is in the preimage of L . $\Rightarrow x_1^T l = 0$
 \exists a line L' through p_1 with coimage $r \neq l$. $\Rightarrow x_1^T r = 0$.
 $\Rightarrow x_1 \sim \hat{l}r$.
- iv. $x_2 \sim \hat{l}s$: analog to iii.

2. $\text{rank} \begin{pmatrix} \hat{x}_1 \Pi_1 \\ \hat{x}_2 \Pi_2 \end{pmatrix} \leq 3$

$$\Rightarrow \exists X \in \mathbb{R}^4 \setminus \{0\} \text{ with } \begin{pmatrix} \hat{x}_1 \Pi_1 \\ \hat{x}_2 \Pi_2 \end{pmatrix} X = 0.$$

$$\Rightarrow \hat{x}_1 \Pi_1 X = 0 \quad \wedge \quad \hat{x}_2 \Pi_2 X = 0,$$

$$\Rightarrow x_1 \times \Pi_1 X = 0 \quad \wedge \quad x_2 \times \Pi_2 X = 0.$$

$\Rightarrow x_1$ and $\Pi_1 X$ are linearly dependent; and x_2 and $\Pi_2 X$ are linearly dependent.

$$\Rightarrow \exists \lambda_1, \lambda_2 \in \mathbb{R} \text{ with } \Pi_1 X = \lambda_1 x_1 \quad \wedge \quad \Pi_2 X = \lambda_2 x_2$$

$\Rightarrow x_1$ and x_2 are projections of X .

$$3. \exists \lambda \in \mathbb{R} : [R', T'] = \lambda [R, T] H = \lambda [R, T] \begin{bmatrix} I & 0 \\ v^\top & v_4 \end{bmatrix} = \lambda [R + Tv^\top, Tv_4]$$

$$\begin{aligned} E' &= \hat{T}' R' \\ &= (\widehat{\lambda v_4 \hat{T}}) \cdot (\lambda (R + Tv^\top)) \\ &= \lambda^2 v_4 \hat{T} (R + Tv^\top) \\ &= \lambda^2 v_4 \hat{T} R + \lambda^2 v_4 \underbrace{\hat{T} T}_{=0} v^\top \\ &= \lambda^2 v_4 \hat{T} R \\ &= \lambda^2 v_4 E \quad \text{with } \lambda^2 v_4 \in \mathbb{R} \end{aligned}$$

$$\Rightarrow E' \sim E$$