Multiple View Geometry: Solution Sheet 9

Prof. Dr. Daniel Cremers, David Schubert, Mohammed Brahimi
Computer Vision Group, TU Munich
http://vision.in.tum.de/teaching/ss2019/mvg2019

Exercise: July 10th, 2019

Part I: Theory

1. **Robust Least Squares**

   (a) What situations can you think of where a robust loss function might be needed?
   Missing (i.e. black or white) pixels in the image, dynamic changes of the scene, non-Lambertian surfaces like shiny or transparent objects, (local) changes in lighting conditions, ...

   (b) Write down the weight function for the Huber loss.

$$w_\delta(t) = \begin{cases} 1 & |t| \leq \delta \\ \frac{\delta}{|t|} & \text{else} \end{cases}$$

2. **Optimization Techniques**

   Write down the update step $\Delta\xi$ for each of the following minimization methods:

   (a) Gradient descent, normal least squares,

$$\Delta\xi = -\lambda J^\top \mathbf{r}$$

   (b) Gradient descent, weighted least squares,

$$\Delta\xi = -\lambda J^\top W \mathbf{r}$$

   (c) Gauss-Newton, normal least squares,

$$\Delta\xi = -(J^\top J)^{-1} J^\top \mathbf{r}$$

   (d) Gauss-Newton, weighted least squares,

$$\Delta\xi = -(J^\top W J)^{-1} J^\top W \mathbf{r}$$

   (e) Levenberg-Marquardt, normal least squares,

$$\Delta\xi = -(J^\top J + \lambda \text{diag}(J^\top J))^{-1} J^\top \mathbf{r}$$

   (f) Levenberg-Marquardt, weighted least squares.

$$\Delta\xi = -(J^\top W J + \lambda \text{diag}(J^\top W J))^{-1} J^\top W \mathbf{r}$$

Note : for normal least square, updates are explained in chapter 7.
For weighted least square, we can use a change of variable to write the optimization problem exactly as a normal least square :

$$\sum_i w(r_i)r_i(\xi)^2 = \sum_i (\sqrt{w(r_i)}r_i(\xi))^2 = \sum_i \tilde{r}_i(\xi)^2 = \|\tilde{r}(\xi)\|^2$$

Then to get the new update steps, we just substitute r with $\tilde{r} = \sqrt{W}r$ and J with $\tilde{J} = \sqrt{W}J$ in the previous one.

($\tilde{J}$ is the jacobian of $\tilde{r}$, and $\sqrt{W}$ is the matrix whose components are the square root of the components of W).