Probabilistic Graphical Models in Computer Vision

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Weekly Exercises 1

Room: 02.09.023
Wednesday, 15.05.2019, 12:15 - 14:00

Probability (12+6 Points)

Exercise 1 (6 Points)**.** Siegfried the ornithologist does a study on the green-speckled swallow. Since he has a huge collection of bird photographs he wants to find all images depicting a green-speckled swallow. Due to it's distinctive features it is an easy task for Eduard, Siegfried's friend and computer vision scientist, to program a green-speckled swallow detector that marks all images containing such a bird. Unfortunately the detector does not work perfectly. If the image contains a green-speckled swallow the detector marks it correctly with a chance of 99.5%. If the image does not contain a green-speckled swallow the detector marks it correctly with a chance of 99.3%. The bird is also very rare: If we randomly draw an image from the collection, there is only a chance of 0.001% that the image contains a green-speckled swallow.

1. Do a formal modeling of the experiment. What is the discrete probability space?
2. What is the probability that a green-speckled swallow is on a given image, if the detector gives a positive answer?
3. What is the probability that a green-speckled swallow is on a given image, if the detector gives a negative answer?

Exercise 2 (4 Points)**.** Assume that A, B and C are events. Assuming $P(B \mid C) \neq 0$, prove that

$$P(A \mid B \cap C) = \frac{P(B \mid A \cap C)P(A \mid C)}{P(B \mid C)}. \tag{1}$$

Exercise 3 (4 Points)**.** Let A, B and C be events and $P(B \cap C) > 0$, show that

$$P(A \mid C) = P(A \mid B \cap C) \;\Leftrightarrow\; P(A \cap B \mid C) = P(A \mid C)P(B \mid C). \tag{2}$$

Exercise 4 (4 Points)**.** If X and Y are independent, show following equation:

$$\mathrm{Cov}[X, Y] = 0$$

Is the converse true? Prove it or give a counter example depending on your answer.