

## Weekly Exercises 2

Room: 02.09.023

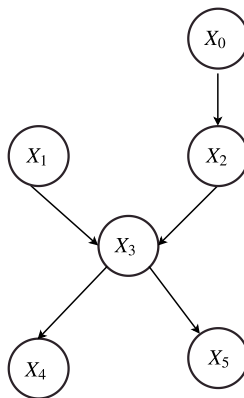
Wednesday, 22.05.2019, 12:15 - 14:00

### Bayesian Network

(12+6 Points)

**Exercise 1** (4 Points). Given two boolean random variables  $X$  and  $Y$ , (*i.e.* each can only be true (1) or false (0)). Show that if  $X = 0$  is independent to  $Y = 0$ , we have  $X$  and  $Y$  are independent.

**Exercise 2** (4 Points). In the following Bayesian network:



1. Give the factorization of  $p(x_0, x_1, x_2, x_3, x_4, x_5)$ .
2. Assume the observation is  $\{X_4\}$ , give reachable nodes of  $\{X_0\}$  via active trail.
3. Assume the observation is  $\{X_0\}$ , give reachable nodes of  $\{X_2\}$  via active trail.

**Exercise 3** (4 Points). Given three boolean random variables  $X$ ,  $Y$  and  $Z$ , is it possible to find a perfect map for following distribution:

$$p(x, y, z) = \begin{cases} \frac{1}{12} & x \oplus y \oplus z = \text{false} \\ \frac{1}{6} & x \oplus y \oplus z = \text{true} \end{cases} \quad (1)$$

where  $\oplus$  is the XOR function. Explain why or draw the corresponding perfect map.

**Exercise 4** (6 Points). For a directed graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , assume that the max indegree is 2 (*i.e.* any node has maximum 2 parents). Given an observation set  $Z$  and a random variable  $Y$ , figure out an algorithm to find the reachable nodes of  $Y$  via active trail in this graph.

Hint: First of all, assume there is no v-structure in the graph, what should we do? Then include the v-structure, what kind of preprocessing do we have to do?

## Programming (Due:27.05)

(12Points)

**Exercise 5.** In this programming exercise, you are asked to implement an algorithm to find reachable nodes via active trail in a directed graph. See the ipython file for more details.