Probabilistic Graphical Models in Computer Vision

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Weekly Exercises 3

Room: 02.09.023

Wednesday, 29.05.2019, 12:15 - 14:00

Markov Random Field (12+6 Points)

Exercise 1 (6 Points)**.** Given two Bayesian network graphs G_1 and G_2, explain following statements for G_1 and G_2 are I-equivalent.

1. "G_1 and G_2 has the same skeleton" is a necessary but not sufficient condition.
2. "G_1 and G_2 has the same skeleton and same v-structure" is a sufficient but not necessary condition.

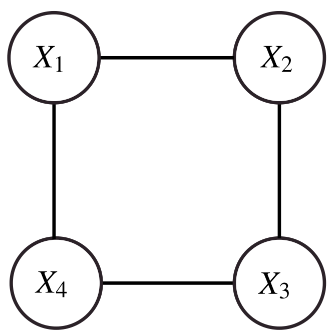

Figure 1: circle graph

Exercise 2 (6 Points)**.** In this problem, we consider a distribution which is not strictly positive and it can not be factorized by a MRF.

Assume we have four binary random variables X_i, $i \in \{1, 2, 3, 4\}$. The probability distribution assigns a probability 1/8 uniformly to each of the following set of values (X_1, X_2, X_3, X_4):

$$\begin{array}{cccc} (0,0,0,0) & (1,0,0,0) & (1,1,0,0) & (1,1,1,0) \\ (0,0,0,1) & (0,0,1,1) & (0,1,1,1) & (1,1,1,1) \end{array}$$

and assigns zero to all other configurations of (X_1, X_2, X_3, X_4).

1. We first show that $p(X_1 = x_1, X_2 = x_2, X_3 = x_3, X_4 = x_4) = p(X_1 = x_4, X_2 = x_3, X_3 = x_2, X_4 = x_1)$.

2. Show that the distribution satisfies the global independencies with respect to the circle graph in Figure 1.
   Hint: show that whatever pair of values for $\{X_2, X_4\}$, the value of X_1 or X_3 is known. Then use the conclusion from previous sub problem.

3. Show that we cannot find any factorization for $p(x_1, x_2, x_3, x_4)$.
   Hint: Try to find a contradiction by examing all $\phi_{ij}(x_i, x_j)$ with (i, j) are the edges in circle graph in Figure 1.

Exercise 3 (6 Points)**.** Let G be a factor graph for a Markov random field consisting of N^2 binary variables, representing the pixels of an $N \times N$ image. For each piexel there is unary potential, and there are pairwise potentials according to the 8-connected neighborhood.

1. Draw the factor graph for $N = 3$.

2. What is the total number of factors, depending on N, that are included in this model.