Probabilistic Graphical Models in Computer Vision

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Weekly Exercises 4

Room: 02.09.023
Wednesday, 05.06.2019, 12:15 - 14:00

Markov Random Field (12+6 Points)

Exercise 1 (4 Points)**.** Firstly, draw one possible factor graph for each Markovf Random Fields shown as following. Then write the corresponding factorization and independence of following 4 Markov Random Fields:

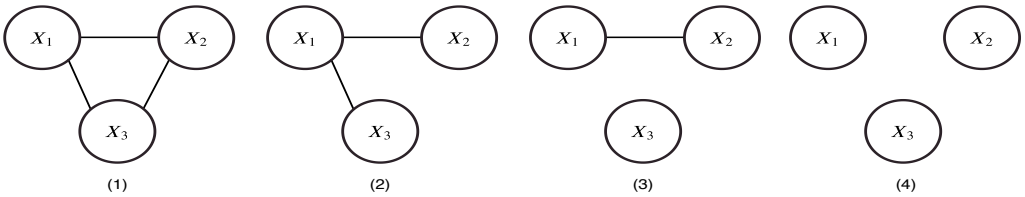
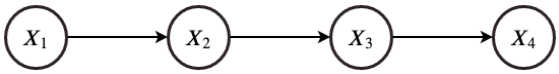


Figure 1: Exercise 1

Exercise 2 (4 Points)**.** Consider following directed chain of 4 random variables, each variable can take n number of values. Assume we want to evaluate the probability that X_4 takes on value x_4, *i.e.* $P(X_4 = x_4)$.



1. If we use straightforward probabilistic description, how many entries are required for a full joint probability table regarding n? How many operations are required ragarding n? (Use Big O notation).

2. Write down the factorization of $P(x_1, x_2, x_3, x_4)$ and explain how we can use it to simplify the computation.

3. How many operations do we need now regarding n? (Use Big O notation).

Exercise 3 (4 Points)**.** Using the same idea from previous exercise, consider following problem, assume each variable has n number of values:

1. Using variable elimination with the order x_1, x_2, x_4, x_3 to compute $P(x_5)$. How many operations do we need regarding n?

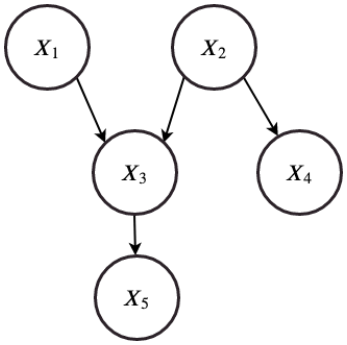


Figure 2: Exercise 3

2. What if we eliminate with the order x_3, x_1, x_2, x_4 to compute $P(x_5)$? How many operations do we need reagarding n?

Exercise 4 (6 Points)**.** Here we use a simple factor graph to practice Belief Propagation. Consider following factor graph: assume designate node x_3 as the root, use

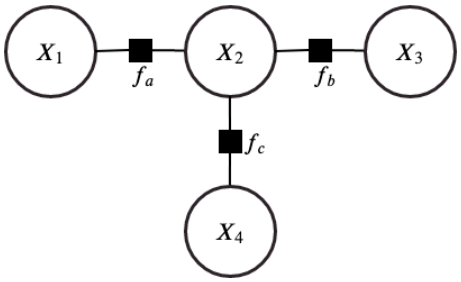


Figure 3: Exercise 4

the Belief Propagation to verify $\tilde{P}(x_2) \propto \sum_{x_1,x_3,x_4} \tilde{P}(x_1, x_2, x_3, x_4)$, where $\tilde{P}$ is the unnormalized probability.