Chair of Computer Vision and Artificial Intelligence
Department of Informatics
Technical University of Munich

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Registration number

Signature

Note:
- Cross your Registration number(with leading zero). It will be evaluated automatically.
- Do not sign the above signature field. A signature field will be provided as part of the first problem.

Computer Vision II: Multiple View Geometry

Exam:	IN2228 / Midterm	**Date:**	Thursday 16th July, 2020
Examiner:	Florian Bernard	**Time:**	14:00 – 14:20

Working instructions

- This exam consists of **8 pages** with a total of **6 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 25 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - this is an open book graded exercise
- This graded exercise uses the TUMexam platform which offers a student manual on their webpage[1].
- The boxes on the sides of the subproblems (those with numbers) are used for correction and ticking them is prohibited. A ticked box that is not part of a multiple choice question may result in zero points for the problem.
- In the multiple choice questions there is always exactly one correct answer. A correct answer (comprising of exactly one tick at the correct position) will give the indicated credits, whereas a wrong answer will result in 0 credits.

  Mark correct answers with a cross ☒
  To undo a cross, completely fill out the answer option ■
  To re-mark an option, use a human-readable marking ×■

- Remark: We aimed to provide a large variety of the type of problems that may occur in the final/re-take exam. **To obtain the grade bonus it is not necessary to attain all credits**. Do not get discouraged if the time is not sufficient to solve all problems. Start with the problems that you can solve easily and then progress to the harder ones.

Left room from ________ to ________ / Early submission at ________

[1] https://tumexam.de/static/handreichung_submissions_students.pdf

Problem 1 **Personal Information (0 credits)**

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1

a) Please sign the following by entering your full name: "I hereby assure that I solve and submit this exam myself under my own name by only using the allowed tools listed on the first page".

0
1

b) Please enter your matriculation number with leading zero.

Problem 2 Linear Algebra (5 credits)

The solution to problem a) should be given in MATLAB syntax, e.g.,

```
A = [a11, a12, a13, a14; a21, a22, a23, a24; a31, a32, a33, a34]
```

a) Consider the linear spaces

0 1 2 3

$$U = \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\},$$

$$V = \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \\ -1 \end{pmatrix} \right\}.$$

Find a matrix $A \in \mathbb{R}^{3\times 4}$ such that U and V are equal to the range and kernel of A. `A =`

b) Let $B \in \mathbb{R}^{n\times k}, C \in \mathbb{R}^{m\times k}, D \in \mathbb{R}^{m\times n}$ and consider the matrix

0 1 2

$$X = (rank(C^\top DB) + 1)^{-1}\ BC^\top D\,.$$

For **given** n, k, m, write down an expression for the minimal rank of X and the maximal rank of X, as well as a **brief** justification.

Minimal rank of X:

Maximal rank of X:

Problem 3 MATLAB (7 credits)

0
1
2

a) Consider the matrix A and vector b given by

$$A = \begin{pmatrix} 4 & 6 \\ 9 & 2 \\ 7 & 2 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 8 \\ 6 \end{pmatrix}.$$

Write a MATLAB one-line expression for `x` that **for the specific** A, b gives a near-identical result to `x_ref = A \ b`, i.e.

```
assert(norm(x-x_ref) < 1e-12)
```

does not throw an error. Your expression may use only the following MATLAB symbols/functions `A, b, *, +, -, inv, ', rank, zeros, eye, ones, magic` and parenthesis. `x =`

0
1
2
3
4
5

b) Let w be a vector in $\mathbb{R}^3$ which is represented in MATLAB as a column vector `w` s.t. `size(w)` gives `[3, 1]`. Let R be a rotation matrix in $SO(3)$ which is represented in MATLAB by `R`. Given are a large number of vectors as the rows of the matrix X in $\mathbb{R}^{100\,000 \times 3}$ which is represented in MATLAB by `X`. Given are `programA`

```
function Y = programA(w, R, X)
    Y = zeros(size(X));
    for i = 1:size(X, 1)
        Y(i, :) = cross(w, (R * X(i,:)'))';
    end
end
```

and `programB`

```
function Y = programB(w, R, X)
    Y = cross(repmat(w', size(X, 1), 1), X * R');
end
```

which are called like `Y_A = programA(w, R, X); Y_B = programB(w, R, X)`.

Give an explanation of what is computed, i.e what is the mathematical relationship between Y_A, Y_B and the inputs w, R, X. What is the relationship between `Y_A` and `Y_B`? What do the rows of `Y_A` and `Y_B` contain?

Which of the two programs is significantly faster? Explain your choice.[2]

Write a one-line expression which computes the same result even faster, i.e. at least a factor of three in median run-time over 10 runs.[3] You are given `w_hat` which is the 3×3 matrix corresponding to $\hat{w}$. Your expression may use only the following MATLAB symbols/functions `R, X, w, w_hat, *, +, -, inv, ', rank, zeros, eye, ones, magic` and parenthesis. `Y_C =`

[2]Assume execution on a standard desktop PC with an i5 CPU and without GPU. You do not have to use this fact in your explanation.
[3]This subproblem gives few points in comparison to the rest of the graded exercise.

Problem 4 Image Formation (4 credits)

A 3D point $P = (2, 0, 4)^\top$ is observed by a camera, which has its optical center at $C = (0, -1, 0)^\top$ and no rotation ($R = \mathrm{Id}_{3\times3}$). The intrinsic parameter matrix K is given by

$$K = \begin{pmatrix} 500 & 0 & 320 \\ 0 & 400 & 240 \\ 0 & 0 & 1 \end{pmatrix}.$$

Calculate the pixel-position of the projected point in the image and tick the correct answer.

a)

- ☐ u = 430
- ☐ u = 260
- ☐ u = 620
- ☐ u = 750
- ☐ u = 770
- ☐ u = 950
- ☐ u = 200
- ☐ u = 570

b)

- ☐ v = 340
- ☐ v = 950
- ☐ v = 650
- ☐ v = 860
- ☐ v = 350
- ☐ v = 810
- ☐ v = 490
- ☐ v = 670

c) Given that the camera is nearly perfect what is the size of the image?

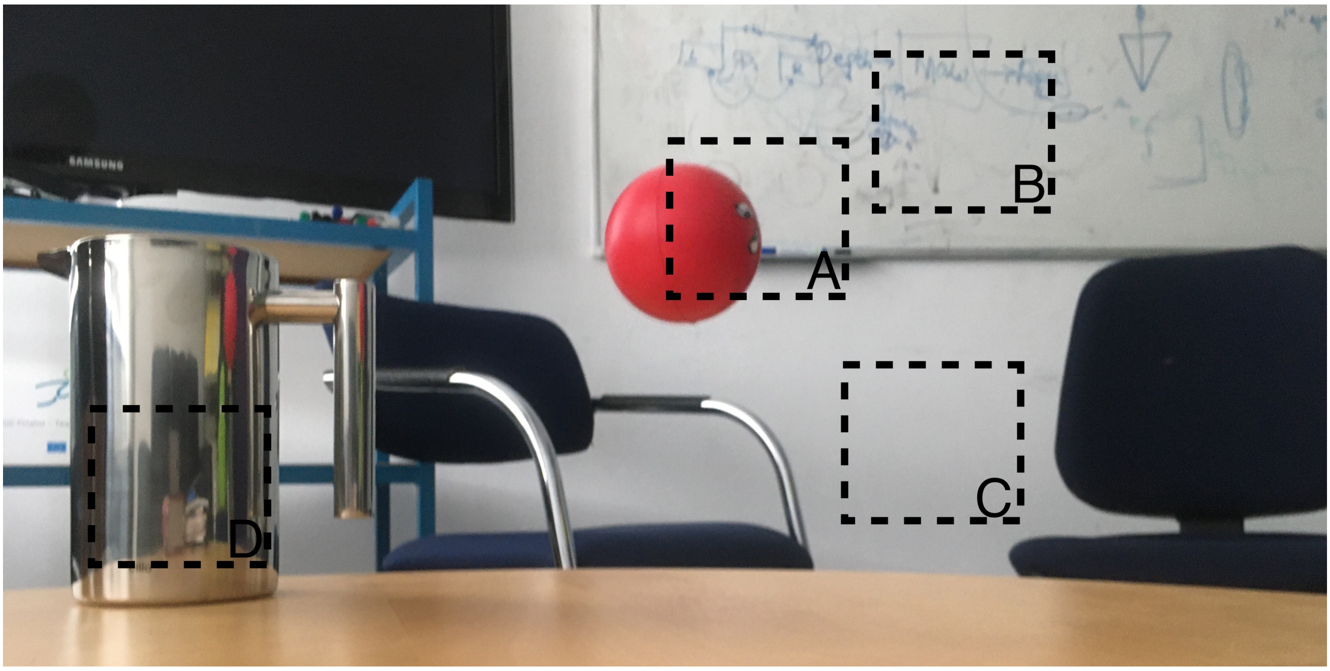
- ☐ height = 800 px, width = 1000 px
- ☐ height = 480 px, width = 640 px
- ☐ height = 240 px, width = 320 px
- ☐ height = 400 px, width = 500 px

d) **Briefly** justify your answer from c).

☐ 0
☐ 1

Problem 5 The Lucas-Kanade Method (4 credits)

Given is the following image where the black, dashed boxes are annotations used within this exercise.



The image is the first image of an image pair used within the Lucas-Kanade algorithm. Assume that the second image is taken 0.2 seconds later. Furthermore assume that the camera motion is slow and that the translation and rotation can be in any direction. The red ball in A is falling. The Lucas-Kanade algorithm is used to estimate the optical flow for the two images. For the marked image regions A – D decide and explain if the algorithm will yield a good result. The neighborhood $W(x)$ is set to be the same as the annotation boxes. Use **precise, technical** terms from the lecture for your explanations.

0
1
a) Image region A:

0
1
b) Image region B:

0
1
c) Image region C:

0
1
d) Image region D:

Problem 6 Fundamental Matrix (5 credits)

Let $F \in \mathbb{R}^{3\times 3}$ be the fundamental matrix for the cameras C_1 and C_2. Let K be the intrinsic camera matrix for C_1, C_2 and let T be the translation between the two camera centers as seen in the coordinate system of camera C_2. Let e_2 be the epipole in the second image.

This exercise will prove that $e_2^\top F = 0$ holds. **Below** each step there will be a solution box in which you should give a **brief** explanation of why the step is correct. Please **briefly** reference the appropriate material from the lecture or the tutorials. Basic mathematical facts do not need to be referenced.

a) 0 1

$$e_2^\top F = (\frac{1}{\lambda_1} KT)^\top (K^{-\top} \hat{T} R K^{-1})$$

b) 0 1

$$= \frac{1}{\lambda_1} T^\top K^\top K^{-\top} \hat{T} R K^{-1}$$

c) 0 1

$$= \frac{1}{\lambda_1} T^\top \hat{T} R K^{-1}$$

d) 0 1

$$= -\frac{1}{\lambda_1} (\hat{T} T)^\top R K^{-1}$$

e) 0 1

$$= 0$$