Multiple View Geometry: Exercise Sheet 8

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Part I: Theory

Download the ICRA 2013 paper *Robust Odometry Estimation for RGB-D Cameras* by Kerl, Sturm and Cremers from the *Publications* sections on our webpage.[1] Read the paper and focus in particular on *III. Direct Motion Estimation*.

1. **Image Warping**

   (a) Look at the warping function $\tau(\xi, \mathbf{x})$ in Eq. (9). What do $\tau(\xi, \mathbf{x})$ and $r_i(\xi)$ look like at $\xi = \mathbf{0}$?

   (b) Prove that the derivative of $r_i(\xi)$ w.r.t. ξ at $\xi = \mathbf{0}$ is

$$\left.\frac{\partial r_i(\xi)}{\partial \xi}\right|_{\xi=\mathbf{0}} = \frac{1}{z}\begin{pmatrix} I_x f_x & I_y f_y \end{pmatrix} \left.\begin{pmatrix} 1 & 0 & -\frac{x}{z} & -\frac{xy}{z} & z + \frac{x^2}{z} & -y \\ 0 & 1 & -\frac{y}{z} & -z - \frac{y^2}{z} & \frac{xy}{z} & x \end{pmatrix}\right|_{(x,y,z)^\top = \pi^{-1}(\mathbf{x}_i, Z_1(\mathbf{x}_i))}$$

   To this end, apply the chain rule multiple times and use the following identity:

$$\left.\frac{\partial T\left(g(\xi), \mathbf{p}\right)}{\partial \xi}\right|_{\xi=\mathbf{0}} = \begin{pmatrix} \mathrm{Id}_3 & -\hat{\mathbf{p}} \end{pmatrix} \in \mathbb{R}^{3\times 6}\,.$$

   Note: The notation $\partial f(x)/\partial x$ denotes the Jacobian matrix including all first-order partial derivatives, where the number of rows is the number of dimensions of $f(x)$, and the number of columns is the number of dimensions of x.

   (c) Following the derivation in (b), determine the derivative for arbitrary ξ

$$\left.\frac{\partial r_i(\Delta\xi \circ \xi)}{\partial \Delta\xi}\right|_{\Delta\xi=\mathbf{0}}$$

   where $\circ$ is defined by

$$\xi_1 \circ \xi_2 := \log\Big(\exp(\widehat{\xi_1}) \cdot \exp(\widehat{\xi_2})\Big)^\vee\,.$$

   $\vee\colon \mathfrak{se}(3) \to \mathbb{R}^6$ is the inverse of the hat transform.
   Hint: Rewrite the problem such that you can make use of part b).

[1] http://vision.in.tum.de/publications

2. **Image Pyramids**

   In order to handle large translational and rotational motions, a coarse-to-fine scheme is applied in the paper. To go from one level l to $l+1$, the images $I^{(l)}$ (intensity) and $D^{(l)}$ (depth) are downscaled by averaging over intensities or valid depth values, respectively:

$$\begin{aligned} I^{(l+1)}(n,m) &:= \frac{1}{4} \cdot \sum_{(n',m') \in O(n,m)} I^{(l)}(n',m') \\ O(n,m) &= \{(2n,2m),(2n+1,2m),(2n,2m+1),(2n+1,2m+1)\} \\ D^{(l+1)}(n,m) &:= \frac{1}{|O_d(n,m)|} \cdot \sum_{(n',m') \in O_d(n,m)} D^{(l)}(n',m') \\ O_d(n,m) &= \{(n',m') \in O(n,m) : D(n',m') \neq 0\} \end{aligned}$$

   How does the camera matrix K change from level l to $l+1$? Write down $f_x^{(l+1)}$, $f_y^{(l+1)}$, $c_x^{(l+1)}$ and $c_y^{(l+1)}$ in terms of $f_x^{(l)}$, $f_y^{(l)}$, $c_x^{(l)}$ and $c_y^{(l)}$.

3. **Optimization for Normally Distributed** $p(r_i)$

   (a) Confirm that a normally distributed $p(r_i)$ with a uniform prior on the camera motion leads to normal least squares minimization. To this end, use

$$p(r_i|\xi) = p(r_i) = A \exp\left(-\frac{r_i^2}{\sigma^2}\right)$$

   to show that with a constant prior $p(\xi)$, the maximum a posteriori estimate is given by

$$\xi_{\text{MAP}} = \arg\min_{\xi} \sum_i r_i(\xi)^2 \,.$$

   (b) Explicitly show that the weights

$$w(r_i) = \frac{1}{r_i} \frac{\partial \log p(r_i)}{\partial r_i}$$

   are constant for normally distributed $p(r_i)$.

   (c) Show that in the case of normally distributed $p(r_i)$ the update step $\Delta\xi$ can be computed as

$$\Delta\xi = -\left(J^\top J\right)^{-1} J^\top \mathbf{r}(\mathbf{0}) \,.$$

Part II: Practical Exercises

In this exercise you will implement direct image alignment as Gauss-Newton minimization on $SE(3)$. Download the package `ex8.zip` provided on the website. It contains a code framework, test images and the corresponding camera calibration.

1. Implement the function `[Id,Dd,Kd] = downscale(I,D,K,level)` which (recursively) halves the image resolution of the image I, the depth map D and adjusts the corresponding camera matrix K per pyramid level l. For an input frame of dimensions 640×480 ($l = 1$), level 2 corresponds to 320×240 pixels, level 3 corresponds to 160×120 pixels and so on. Use the equations and results obtained in the theory part.

2. Complete the function `r = calcErr(I1, D1, I2, xi, K)` that takes the images and their (assumed) relative pose, and calculates the per-pixel residual $\mathbf{r}(\xi)$ as defined in the slides. `r` should be a $n \times 1$ vector, with $n = w \times h$, the number of pixels. Visualize the residual as an image for $\xi = \mathbf{0}$.
   Hint: perform tests on a coarse version of the image (e.g. 160×120*) to make it run faster.*

3. Implement the function `[J, r] = deriveNumeric(I1, D1, I2, xi, K)` that differentiates $\mathbf{r}(\Delta\xi \circ \xi)$ **numerically** w.r.t. $\Delta\xi$: for each pixel $\mathbf{x}_i$ compute

$$\left.\frac{\partial r_i(\Delta\xi \circ \xi)}{\partial \Delta\xi}\right|_{\Delta\xi=0} \approx \left(\frac{r_i((\epsilon\mathbf{e}_1) \circ \xi) - r_i(\xi)}{\epsilon}, \ldots, \frac{r_i((\epsilon\mathbf{e}_6) \circ \xi) - r_i(\xi)}{\epsilon}\right)$$

   where ϵ is a small value (for Matlab $\epsilon = 10^{-6}$), $\mathbf{e}_j$ is the j'th unit vector and the operator $\circ$ is defined as in exercise 1.(c) of Part I. `J` should be a $n \times 6$ matrix. The per-pixel residuals $\mathbf{r}(\xi)$ are returned as `r`.

4. Implement Gauss-Newton minimization for the photometric error

$$E(\xi) = \sum_i r_i(\xi)^2 = ||\mathbf{r}(\xi)||_2^2$$

   according to the theory part. To this end, complete the script `ex08` in ll. 70 and ll. 75. For an update $\Delta\xi$, compute the updated motion as $\xi_{\text{new}} = \Delta\xi \circ \xi_{\text{old}}$. Use only one pyramid level $l = 3$ (160×120) in the beginning, and then add the others.

5. Implement a function `J = deriveAnalytic(I1, D1, I2, xi, K)` that differentiates $\mathbf{r}(\Delta\xi \circ \xi)$ **analytically** w.r.t. $\Delta\xi$. Use the result of the theory part, Exercise 1 (c). The use of this analytical gradient instead of the numeric derivatives in the minimization should result in a significant speed-up.

6. Run your implementation on the provided images using the script `ex08`.