Multiple View Geometry: Solution Sheet 9

Prof. Dr. Florian Bernard, Lukas Köstler, Mohammed Brahimi
Computer Vision Group, TU Munich
`http://vision.in.tum.de/teaching/ss2020/mvg2020`

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Part I: Theory

1. **Robust Least Squares**

    (a) What situations can you think of where a robust loss function might be needed?
    Missing (i.e. black or white) pixels in the image, dynamic changes of the scene, non-Lambertian surfaces like shiny or transparent objects, (local) changes in lighting conditions, ...

    (b) Write down the weight function for the Huber loss.

    $$w_\delta(t) = \begin{cases} 1 & |t| \leq \delta \\ \frac{\delta}{|t|} & \text{else} \end{cases}$$

2. **Optimization Techniques**
    Write down the update step $\Delta\xi$ for each of the following minimization methods:

    (a) Gradient descent, normal least squares,

    $$\Delta\xi = -\lambda J^\top \mathbf{r}$$

    (b) Gradient descent, weighted least squares,

    $$\Delta\xi = -\lambda J^\top W \mathbf{r}$$

    (c) Gauss-Newton, normal least squares,

    $$\Delta\xi = -(J^\top J)^{-1} J^\top \mathbf{r}$$

    (d) Gauss-Newton, weighted least squares,

    $$\Delta\xi = -(J^\top W J)^{-1} J^\top W \mathbf{r}$$

    (e) Levenberg-Marquardt, normal least squares,

    $$\Delta\xi = -(J^\top J + \lambda \text{diag}(J^\top J))^{-1} J^\top \mathbf{r}$$

    (f) Levenberg-Marquardt, weighted least squares.

    $$\Delta\xi = -(J^\top W J + \lambda \text{diag}(J^\top W J))^{-1} J^\top W \mathbf{r}$$

Note : for normal least square, updates are explained in chapter 7.

For weighted least square, we can use a change of variable to write the optimization problem exactly as a normal least square :

$$\sum_i w(r_i) r_i(\xi)^2 = \sum_i (\sqrt{w(r_i)} r_i(\xi))^2 = \sum_i \tilde{r}_i(\xi)^2 = \|\tilde{r}(\xi)\|^2$$

Then to get the new update steps, we just substitute r with $\tilde{r} = \sqrt{W} r$ and J with $\tilde{J} = \sqrt{W} J$ in the previous one.

($\tilde{J}$ is the jacobian of $\tilde{r}$, and $\sqrt{W}$ is the matrix whose components are the square root of the components of W).