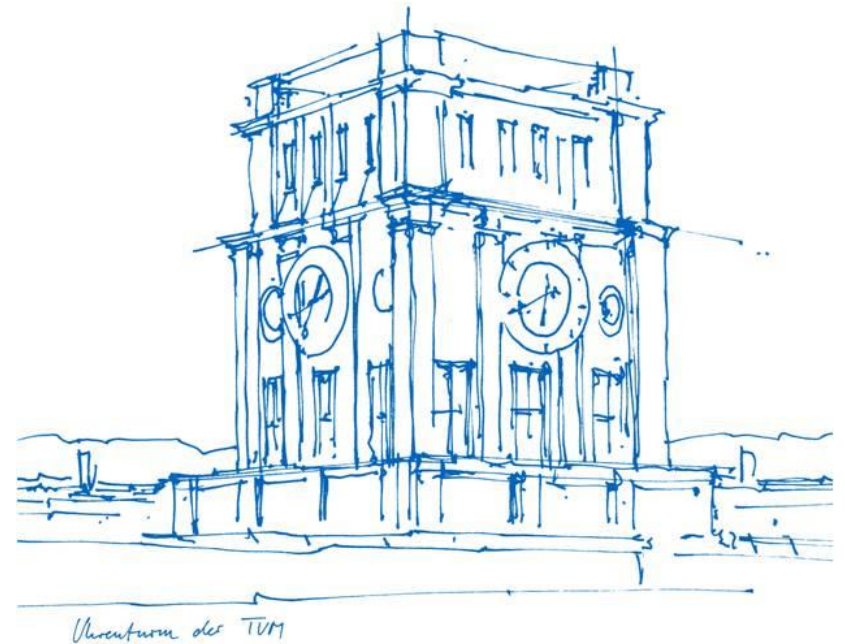


Robust Odometry Estimation for RGB-D Cameras

Kerl, Sturm, Cremers 2013

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Goals and Purpose



[1]

Odometry: Tracking the pose of a robot (position and orientation)

Purpose

- Visual odometry important: no external reference system available (e.g GPS)
- Position control of quadrocopters (indoor)

Goal

Develop method that

- estimates motion of a RGB-D camera
- has high frame rates, low latency and is robust to outliers
- uses these estimates for local navigation and position control (of a quadrocopter)

State of the Art

Real-time visual odometry from dense RGB-D images [2]

- Minimize photo consistency error (energy formulation)
 - Linearization of objective
 - Assumes static scene
- Real-time at 12.5 Hz (different hardware)

Kinect Fusion [3]

- Uses only depth map values
 - Global model of the environment using ICP algorithm
- Only applicable for small indoor rooms (memory issues)
- GPU for real-time (update surface model + process new input)

Contributions

Probabilistic formulation for direct motion estimation (first time)

Robust sensor model from real world data

Integration of temporal prior

Runs in real-time (30 Hz) on a single CPU core (open-source implementation provided)

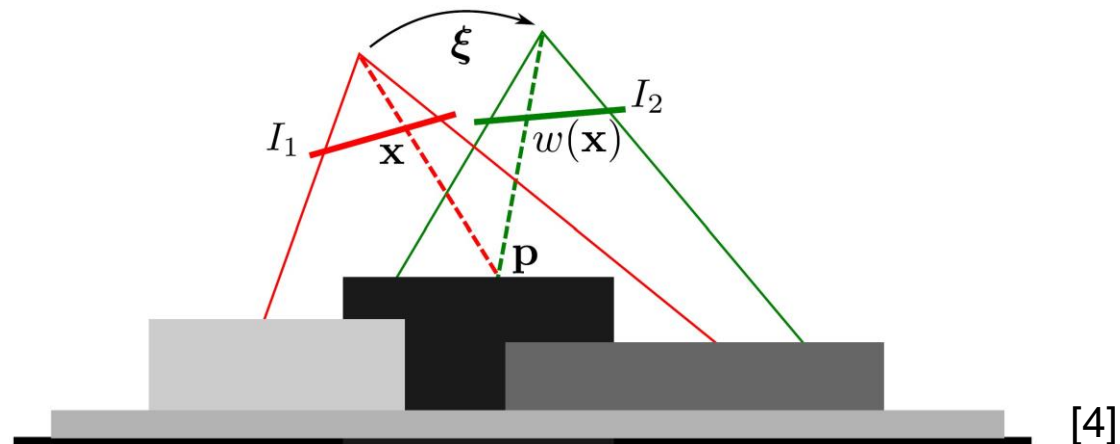
Demonstrating robustness of approach on several benchmarks

Direct Motion Estimation

Based on photo consistency-assumption:

$$I_1(x) = I_2(\tau(\xi, x))$$

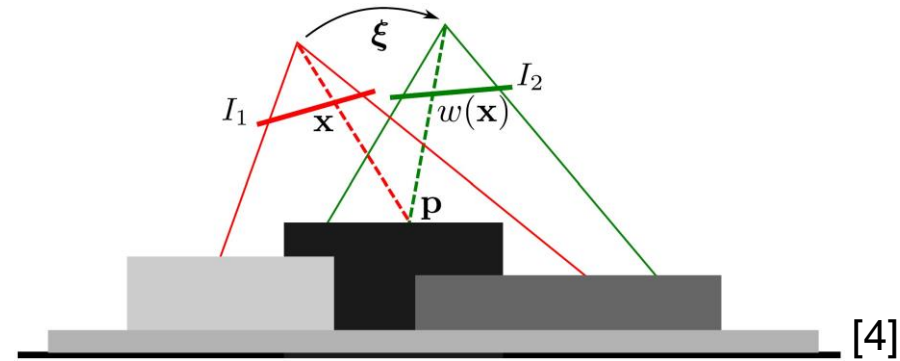
→ Goal: Find ξ (camera motion) that yields the best photo-consistency for all pixels



[4]

Warping Function

$$3\text{D-point } \mathbf{p} = (x, y, z)^T \begin{matrix} \xrightarrow{\pi} \\ \xleftarrow{\pi^{-1}} \end{matrix} \text{Pixel } \mathbf{x} = (u, v)^T$$



Projection function π

$$\mathbf{x} = (u, v)^T = \pi(\mathbf{p}) = \left(\frac{f_x x}{z} + c_x, \frac{f_y y}{z} + c_y \right)^T$$

$$\mathbf{p} = \pi^{-1}(\mathbf{x}, Z_1(\mathbf{x})) = Z_1(\mathbf{x}) \cdot \left(\frac{u - c_x}{f_x}, \frac{v - c_y}{f_y}, 1 \right)^T$$

Point \mathbf{p} viewed in second frame (\mathbf{p}')

$$\mathbf{p}' = R\mathbf{p} + \mathbf{t}$$

Twist coordinates $\xi \in \mathbb{R}^6$

$$\exp(\hat{\xi}) = \begin{pmatrix} R & \mathbf{t} \\ \mathbf{1} & 0 \end{pmatrix}$$

→

$$\tau(\xi, \mathbf{x}) = \pi(R\mathbf{p} + \mathbf{t}) \text{ with } \mathbf{p} = \pi^{-1}(\mathbf{x}, Z_1(\mathbf{x}))$$

Likelihood Function

Residual of the i_{th} pixel

$$r_i(\xi) = I_2(\tau(\xi, \mathbf{x}_i)) - I_1(\mathbf{x}_i)$$

Residual image \mathbf{r}

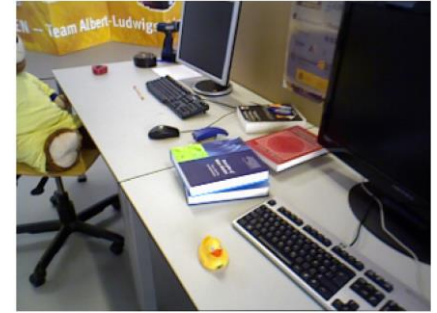
$$p(\mathbf{r}|\xi) = \prod_i p(r_i|\xi)$$

→ Bayes rule

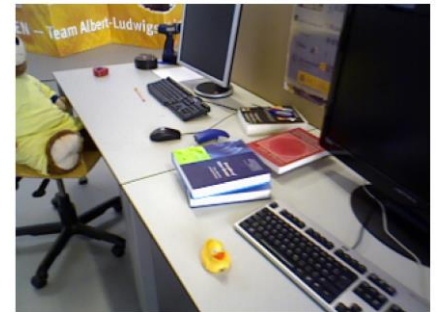
$$p(\xi|\mathbf{r}) = \frac{p(\mathbf{r}|\xi)p(\xi)}{p(\mathbf{r})}$$

Sensor model $p(\mathbf{r}|\xi)$

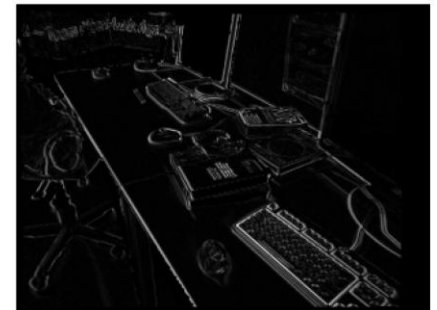
Motion prior $p(\xi)$



(a) reference image I_1



(b) current image I_2



(c) residuals [4]

Maximum A Posteriori (MAP) Estimation

$$\xi_{MAP} = \arg \min_{\xi} -\log p(\xi|\mathbf{r}) = \arg \min_{\xi} -\sum_i \log p(r_i|\xi) - \log p(\xi)$$

Find minimizer (ignore prior for now):

$$\frac{\partial}{\partial \xi} \log p(\xi|\mathbf{r}) \stackrel{!}{=} 0 \rightarrow \sum_i \frac{\partial r_i}{\partial \xi} w(r_i) r_i = 0 \quad \text{with } w(r_i) = \frac{1}{r_i} \cdot \frac{\partial}{\partial r_i} \log p(r_i|\xi)$$

→ Minimizer also minimizes the weighted least squares problem

$$\xi_{MAP} = \arg \min_{\xi} \sum_i w(r_i) (r_i(\xi))^2$$

Solve minimization problem

→ iteratively reweighted least squares algorithm:

Alternate between computation of weights and estimates of ξ until convergence

Linearization

Need to solve:

$$\sum_i \frac{\partial r_i}{\partial \xi} w(r_i) r_i = 0 \quad (I)$$

Problem: r_i non-linear in ξ

→ Linearize ξ :

$$\begin{aligned} r_{lin}(\xi, \mathbf{x}_i) &= r(\mathbf{0}, \mathbf{x}_i) + \left. \frac{\partial r(\tau(\xi, \mathbf{x}_i))}{\partial \xi} \right|_{\xi=\mathbf{0}} \Delta \xi \\ &= r(\mathbf{0}, \mathbf{x}_i) + J_i \Delta \xi \end{aligned}$$

→ Into (I)

$$J^T W J \Delta \xi = -J^T W \mathbf{r}(\mathbf{0})$$

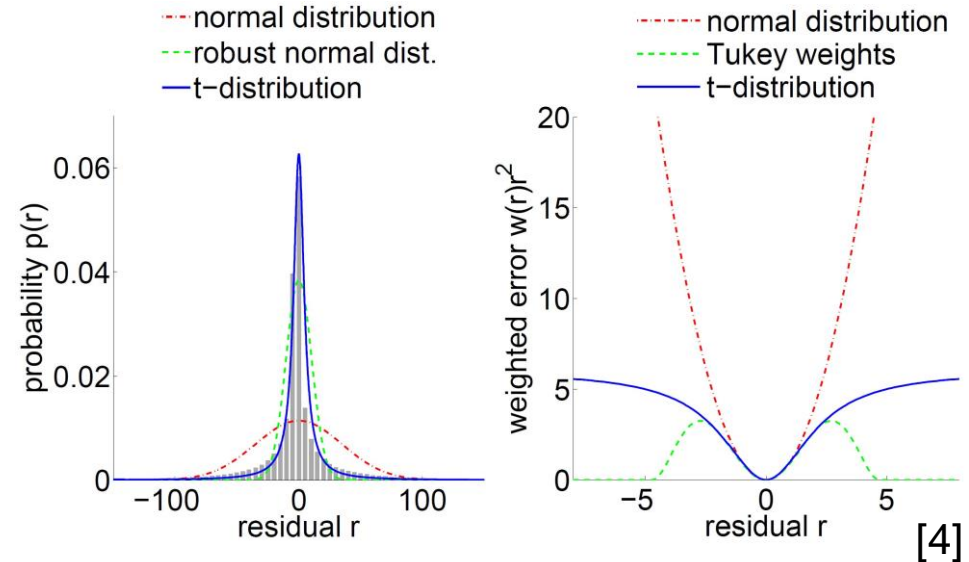
Update parameters:

$$\xi^{(k+1)} = \log(\exp(\xi^{(k)}) \exp(\Delta \xi^{(k)}))$$

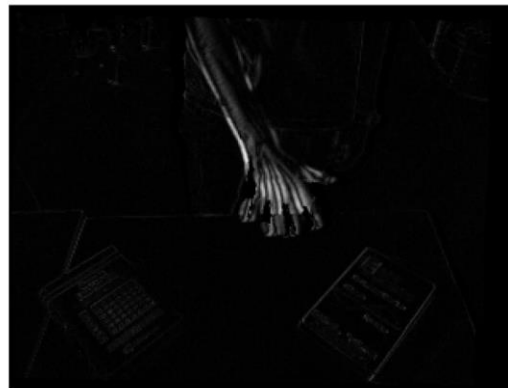
Robust Estimation

Sensor model $p(r|\xi)$

→ t-distribution fits the assumptions better



(a) scene



(b) residuals



(c) weights

Robust Estimation

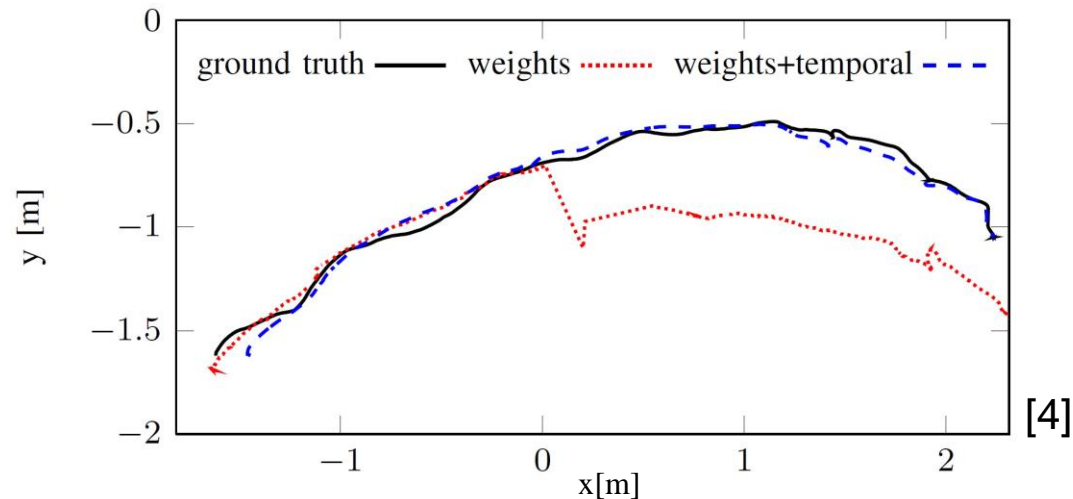
Motion prior $p(\xi)$

Temporal prior based on constant velocity model with a normal distribution:

$$p(\xi_t) = \mathcal{N}(\xi_{t-1}, \Sigma)$$

→ Normal equation:

$$(J^T W J + \Sigma^{-1}) \Delta \xi = -J^T W \mathbf{r}(\mathbf{0}) + \Sigma^{-1} (\xi_{t-1} - \xi_t^{(k)})$$



Evaluation

Synthetic Sequences

- Generated perfect ground truth (two types)
 - Static scene
 - Small dynamic object
- Focus on real-time parameters

Conclusion

- Robust weighting important for noise and outliers
- t-distribution better suited than gaussian distribution (no weights)

static

Method	Parameter set	RMSE [m/s]	Improvement	ØRuntime [s]
reference		0.0751	0.0 %	0.038
no weights	realtime	0.0223	70.0 %	0.032
Tukey weights	realtime	0.0497	33.8 %	0.050
t-dist. weights	realtime	0.0142	81.1 %	0.043

dynamic

Method	Parameter set	RMSE [m/s]	Improvement	ØRuntime [s]
reference		0.1019	0.0 %	0.035
no weights	realtime	0.0650	36.2 %	0.030
Tukey weights	realtime	0.0382	62.5 %	0.056
t-dist. weights	realtime	0.0296	71.0 %	0.047

Evaluation

TUM RGB-D Benchmark Sequences

- Real world data tracked by highly accurate motion capture system
- Focus on real-time parameters

Conclusion

Temporal prior: Particularly useful when many outliers are present or high velocity of camera

static			vs.	dynamic		
Method	ØDrift [m/s]	Improvement		Method	ØDrift [m/s]	Improvement
reference	0.2425	0.0 %		reference	0.0800	0.0 %
no weights	0.0588	75.74 %		no weights	0.0350	56.23 %
Tukey weights	0.1491	38.50 %		Tukey weights	0.1038	-29.66 %
t-dist. weights	0.0432	82.18 %		t-dist. weights	0.0470	41.33 %
t-dist. weights + temporal	0.0428	82.35 %		t-dist. weights + temporal	0.0316	60.55 %

Summary

Probabilistic formulation

- Directly estimate the camera motion from RGB-D images
- Use of custom probability distributions for the sensor and motion model possible

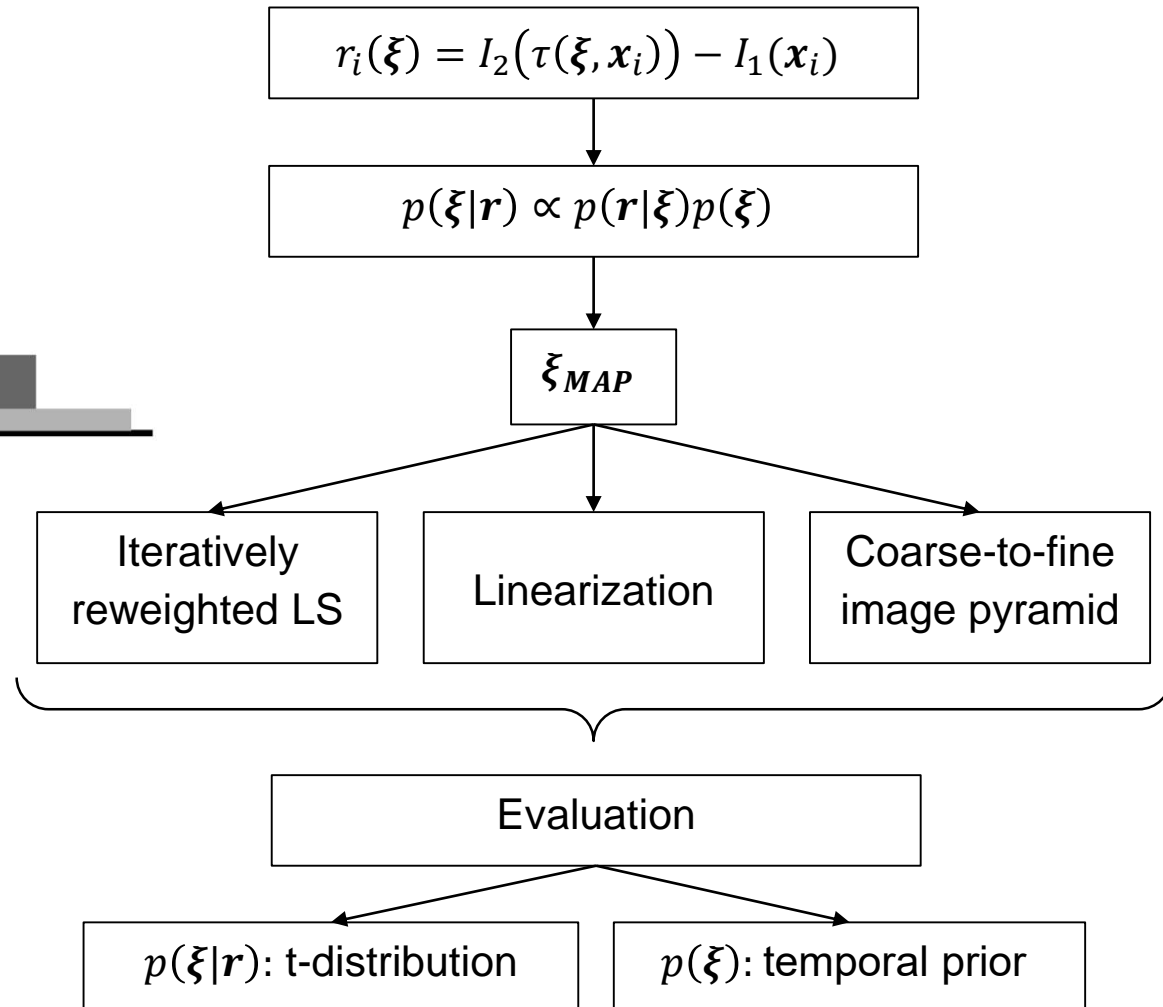
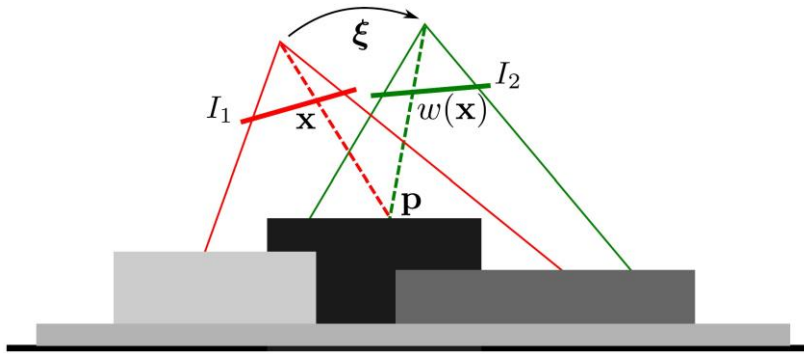
Increase **robustness**

- t-distribution (address outliers)
- Temporal prior (guides optimization)

Fast

Runs in real-time (30 Hz) on a single CPU core

Discussion



References

- [1] <https://www.rcmoment.com/p-rm11238w.html>
- [2] F. Steinbrücker, J. Sturm, and D. Cremers, “Real-time visual odometry from dense RGB-D images,” in *Workshop on Live Dense Reconstruction with Moving Cameras at the Intl. Conf. on Computer Vision (ICCV)*, 2011
- [3] R. A. Newcombe, S. Izadi, O. Hilliges, D. Molyneaux, D. Kim, A. J. Davison, P. Kohli, J. Shotton, S. Hodges, and A. Fitzgibbon, “KinectFusion: Real-time dense surface mapping and tracking,” in *IEEE Intl. Symposium on Mixed and Augmented Reality (ISMAR)*, 2011
- [4] and where no different source is explicitly stated:
C. Kerl, J. Sturm, and D. Cremers, “Robust odometry estimation for RGB-D cameras,” in *IEEE Intl. Conf. on Robotics and Automation (ICRA)*, 2013.

t-distribution

$$p(r) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{r^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

→

$$\begin{aligned} w(r) &= \frac{1}{r} \cdot \frac{\partial}{\partial r} \log p(r) \\ &= \frac{\nu+1}{\nu + \left(\frac{r}{\sigma}\right)^2} \end{aligned}$$

With

$$\sigma^2 = \frac{1}{n} \sum_i r_i^2 \frac{\nu+1}{\nu + \left(\frac{r}{\sigma}\right)^2}$$

(has to be solved iteratively, but converges fast)

Experiments: $\nu = 5$ good

Twist coordinates

$$\xi = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

$$\exp(\hat{\xi}) = \exp \begin{pmatrix} 0 & -\omega_3 & \omega_2 & v_1 \\ \omega_3 & 0 & -\omega_1 & v_2 \\ -\omega_2 & \omega_1 & 0 & v_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\exp(\hat{\xi}) = \begin{pmatrix} R & \mathbf{t} \\ 1 & 0 \end{pmatrix}$$

Video

<https://www.youtube.com/watch?v=TMqPwoCCmto>

Synthetic Sequences

static

Method	Parameter set	RMSE [m/s]	Improvement	ØRuntime [s]
reference		0.0751	0.0 %	0.038
no weights	realtime	0.0223	70.0 %	0.032
Tukey weights	realtime	0.0497	33.8 %	0.050
t-dist. weights	realtime	0.0142	81.1 %	0.043
no weights	precision	0.0145	80.7 %	0.115
Tukey weights	precision	0.0279	62.8 %	0.230
t-dist. weights	precision	0.0124	83.5 %	0.405

dynamic

Method	Parameter set	RMSE [m/s]	Improvement	ØRuntime [s]
reference		0.1019	0.0 %	0.035
no weights	realtime	0.0650	36.2 %	0.030
Tukey weights	realtime	0.0382	62.5 %	0.056
t-dist. weights	realtime	0.0296	71.0 %	0.047
no weights	precision	0.0502	50.7 %	0.130
Tukey weights	precision	0.0270	73.5 %	0.279
t-dist. weights	precision	0.0133	87.0 %	0.508

Real-time visual odometry from dense RGB-D images

Energy formulation

$$E(\xi) = \int I_2(\tau(\xi, \mathbf{x})) - I_1(\mathbf{x})$$

→ Linearize $I_2(\tau(\xi, \mathbf{x}))$ and $\tau(\xi, \mathbf{x})$ around $\xi = \mathbf{0}$

Rigid body motion g

$$E(\xi) \sim \int \left(I_2(\mathbf{x}) - I_1(\mathbf{x}) + \nabla I_2^T \left(\frac{d\pi}{dg} \right) \left(\frac{dg}{d\xi} \right) \xi \right)^2$$

→ Normal equations