Multiple View Geometry: Exercise Sheet 6

Prof. Dr. Daniel Cremers, Simon Weber, Tarun Yenamandra
Computer Vision Group, TU Munich
`Wednesdays 16:00-18:15 at Hörsaal 2, "Interims I" (5620.01.102), and on RBG Live`

Exercise: June 22nd, 2022

Part I: Theory

1. The non-zero essential matrix $E = \hat{T}R$ with $T \in \mathbb{R}^3$ and $R \in \mathrm{SO}(3)$ has the singular value decomposition $E = U\Sigma V^T$. Let $R_Z\left(\pm\frac{\pi}{2}\right)$ be the rotation by $\pm\frac{\pi}{2}$ around the z-axis.

   According to the lecture (Chapter 5, Slide 9), there exist exactly two options for $(\hat{T}, R)$:

$$\left(\hat{T}_1, R_1\right) = \left(UR_Z\left(+\frac{\pi}{2}\right)\Sigma\, U^\top, \quad UR_Z\left(+\frac{\pi}{2}\right)^\top V^\top\right) \tag{1}$$

$$\left(\hat{T}_2, R_2\right) = \left(UR_Z\left(-\frac{\pi}{2}\right)\Sigma\, U^\top, \quad UR_Z\left(-\frac{\pi}{2}\right)^\top V^\top\right) \tag{2}$$

   Show that by using (1) and (2), the following properties hold:

   (a) $\hat{T}_1, \hat{T}_2 \in so(3)$ (i.e. $\hat{T}_1, \hat{T}_2$ are skew-symmetric matrices)

   (b) $R_1, R_2 \in \mathrm{SO}(3)$ (i.e. R_1, R_2 are rotation matrices)

2. Consider the matrices $E = \hat{T}R$ and $H = R + Tu^\top$ with $R \in \mathbb{R}^{3\times3}$ and $T, u \in \mathbb{R}^3$.
   Show that the following holds:

   (a) $E = \hat{T}H$

   (b) $H^\top E + E^\top H = 0$

3. Let $F \in \mathbb{R}^{3\times3}$ be the fundamental matrix for the cameras C_1 and C_2. Show that the following holds for the epipoles e_1 and e_2:

$$Fe_1 = 0 \qquad \text{and} \qquad e_2^\top F = 0$$

Part II: Practical Exercises

1. Download the package `ex06.zip` from the website and extract the images `batinria0.tif` and `batinria1.tif`.
2. **Get the 2D coordinates of corresponding point pairs:** Show the first image and mark at least 8 points. You can retrieve the pixel coordinates of mouse clicks with the command `[x,y] = ginput(gcf)`. Then show the second image and click at the corresponding points in the same order. Again you can get the pixel coordinates with `ginput`. Now you should have the 2D coordinates of corresponding point pairs.
3. **Implement the 8-point algorithm** from the lecture and run it with these point pairs. To this end, you have to transform the coordinates. The intrinsic camera matrices are:

$$K1 = \begin{pmatrix} 844.310547 & 0 & 243.413315 \\ 0 & 1202.508301 & 281.529236 \\ 0 & 0 & 1 \end{pmatrix}$$

$$K2 = \begin{pmatrix} 852.721008 & 0 & 252.021805 \\ 0 & 1215.657349 & 288.587189 \\ 0 & 0 & 1 \end{pmatrix}$$

4. Reconstruct the depths of the points as described in Chapter 5 on Slides 17 and 18.
5. Visualize 3D points and cameras to check whether the results are plausible. You may find the function `plotCamera` useful.

Hints:

- The file `additional_information.txt` provides $K1$ and $K2$
- Use `kron` and `reshape`
- If U or V of the SVD of E have a negative determinant, multiply them by -1, respectively, such that both have a positive determinant.