

Part II: Practical Exercises

1. Homogeneous transformation matrices

- (a) Download the package `ex03.zip` and use `openOFF.m` to load the 3D model `model.off`.
- (b) Write a function that rotates the model around its *center* (i.e. the mean of its vertices) for given rotation angles α , β and γ around the x -, y - and z -axis. Use homogeneous coordinates and describe the overall transformation by a single matrix. The rotation matrices around the respective axes are as follows:

$$\begin{array}{ccc} \text{rotation matrix (x-axis)} & \text{rotation matrix (y-axis)} & \text{rotation matrix (z-axis)} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{pmatrix} & \begin{pmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{pmatrix} & \begin{pmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

Hint: Rotating around the center can be done by translating the center to the origin, then performing the rotation and afterwards translating the center back to its original place. The translations can also be expressed as a matrix multiplication with homogeneous coordinates.

- (c) Rotate the model first 45 degrees around the x -axis and then 120 degrees around the z -axis. Now start again by doing the same rotation around the z -axis first followed by the x -axis rotation. What do you observe?
- (d) Extend the function such that it can perform a translation in addition to the rotation. Find a suitable matrix from $SE(3)$ for this purpose. Translate the model by the vector $(0.5 \ 0.2 \ 0.1)^\top$.

2. Twist-coordinates

- (a) Write a function which takes a vector $w \in \mathbb{R}^3$ as input and returns its corresponding element $R = e^{\hat{w}} \in SO(3) \subset \mathbb{R}^{3 \times 3}$ from the Lie group. Hence, the function will be a concatenation of the hat operator $\hat{\cdot}: \mathbb{R}^3 \rightarrow so(3)$ and the exponential mapping.
- (b) Implement another function which performs the corresponding inverse transformation and test the two functions on some examples.
- (c) Implement similar functions which calculate the transformation for twists. I.e. from $\xi \in \mathbb{R}^6$ to $e^{\hat{\xi}} \in SE(3) \subset \mathbb{R}^{4 \times 4}$ and the other way around.
- (d) How can you use Matlab's built-in functions `expm` and `logm` to achieve the same functionality (your solutions to (a)-(c) should *not* use these functions)?