



# Multiple View Geometry: Solution Sheet 9

Prof. Dr. Daniel Cremers, Simon Weber, Tarun Yenamandra  
Computer Vision Group, TU Munich

Wednesdays 16:00–18:15 at Hörsaal 2, "Interims I"  
(5620.01.102), and on RBG Live

Exercise: July 13th, 2022

---

## Part I: Theory

### 1. Robust Least Squares

- (a) What situations can you think of where a robust loss function might be needed?  
Missing (i.e. black or white) pixels in the image, dynamic changes of the scene, non-Lambertian surfaces like shiny or transparent objects, (local) changes in lighting conditions, ...
- (b) Write down the weight function for the Huber loss.

$$w_{\delta}(t) = \begin{cases} 1 & |t| \leq \delta \\ \frac{\delta}{|t|} & \text{else} \end{cases}$$

### 2. Optimization Techniques

Write down the update step  $\Delta\xi$  for each of the following minimization methods:

- (a) Gradient descent, normal least squares,

$$\Delta\xi = -\lambda J^{\top} \mathbf{r}$$

- (b) Gradient descent, weighted least squares,

$$\Delta\xi = -\lambda J^{\top} W \mathbf{r}$$

- (c) Gauss-Newton, normal least squares,

$$\Delta\xi = -(J^{\top} J)^{-1} J^{\top} \mathbf{r}$$

- (d) Gauss-Newton, weighted least squares,

$$\Delta\xi = -(J^{\top} W J)^{-1} J^{\top} W \mathbf{r}$$

- (e) Levenberg-Marquardt, normal least squares,

$$\Delta\xi = -(J^{\top} J + \lambda \text{diag}(J^{\top} J))^{-1} J^{\top} \mathbf{r}$$

- (f) Levenberg-Marquardt, weighted least squares.

$$\Delta\xi = -(J^{\top} W J + \lambda \text{diag}(J^{\top} W J))^{-1} J^{\top} W \mathbf{r}$$

Note : for normal least square, updates are explained in chapter 7.

For weighted least square, we can use a change of variable to write the optimization problem exactly as a normal least square :

$$\sum_i w(r_i)r_i(\xi)^2 = \sum_i (\sqrt{w(r_i)}r_i(\xi))^2 = \sum_i \tilde{r}_i(\xi)^2 = \|\tilde{r}(\xi)\|^2$$

Then to get the new update steps, we just substitute  $r$  with  $\tilde{r} = \sqrt{W}r$  and  $J$  with  $\tilde{J} = \sqrt{W}J$  in the previous one.

( $\tilde{J}$  is the jacobian of  $\tilde{r}$ , and  $\sqrt{W}$  is the matrix whose components are the square root of the components of  $W$ ).