Multiple View Geometry: Exercise 1

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Math Background

1. Show for each of the following sets (1) whether they are linearly independent, (2) whether they span $\mathbb{R}^3$ and (3) whether they form a basis of $\mathbb{R}^3$:

   (a) $B_1 = \left\{ \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix} \right\}$

   (b) $B_2 = \left\{ \begin{pmatrix}2\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix} \right\}$

   (c) $B_3 = \left\{ \begin{pmatrix}2\\1\\0\end{pmatrix}, \begin{pmatrix}3\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix} \right\}$

2. Which of the following sets forms a group (with matrix-multiplication)? Prove or disprove!

   (a) $G_1 := \left\{ A \in \mathbb{R}^{n\times n} \middle| \det(A) \neq 0 \wedge A^\top = A \right\}$

   (b) $G_2 := \{A \in \mathbb{R}^{n\times n} | \det(A) = -1\}$

   (c) $G_3 := \{A \in \mathbb{R}^{n\times n} | \det(A) > 0\}$

3. Prove or disprove: There exist vectors $\mathbf{v}_1, ..., \mathbf{v}_5 \in \mathbb{R}^3 \setminus \{\mathbf{0}\}$, which are pairwise orthogonal, i.e.
$$\forall i, j = 1, ..., 5 : \quad i \neq j \implies \langle \mathbf{v}_i, \mathbf{v}_j \rangle = 0$$

4. Which groups have you seen in the lecture? Write down the names and the correct inclusions! (e.g.: group A $\subset$ group B)

5. Let A be a symmetric matrix, and λ_a, λ_b eigenvalues with eigenvectors v_a and v_b. Prove: if v_a and v_b are not orthogonal, it follows: $\lambda_a = \lambda_b$.

   Hint: What can you say about $\langle Av_a, v_b \rangle$?

6. Let $A \in \mathbb{R}^{n\times n}$ be a symmetric matrix with the orthonormal basis of eigenvectors $v_1, \ldots, v_n$ and eigenvalues $\lambda_1 \geq \ldots \geq \lambda_n$. Find all vectors x, that minimize the following term:
$$\min_{||x||=1} x^\top A x$$

How many solutions exist? How can the term be maximized?

Hint: Use the expression $x = \sum_{i=1}^{n} \alpha_i v_i$ with coefficients $\alpha_i \in \mathbb{R}$ and compute appropriate coefficients!

7. Let $A \in \mathbb{R}^{m \times n}$. Prove that kernel$(A) =$ kernel$(A^\top A)$.

   Hint: Consider a) $x \in$ kernel$(A) \Rightarrow x \in$ kernel$(A^\top A)$
   and b) $x \in$ kernel$(A^\top A) \Rightarrow x \in$ kernel(A).

8. Singular Value Decomposition (SVD)

   Let $A = USV^\top$ be the SVD of A.

   (a) Write down possible dimensions for A, U, S and V.

   (b) What are the similarities and differences between the SVD and the eigenvalue decomposition?

   (c) What do you know about the relationship between U, S, V and the eigenvalues and eigenvectors of $A^\top A$ and $AA^\top$?

   (d) What is the interpretation of the entries in S and what do the entries of S tell us about A?