Multiple View Geometry: Exercise 4

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The Lucas-Kanade method
The weighted Lucas-Kanade energy $E(\mathbf{v})$ is defined as

$$E(\mathbf{v}) = \int_{W(\mathbf{x})} G(\mathbf{x}-\mathbf{x}') \left\| \nabla I(\mathbf{x}',t)^\top \mathbf{v} + \partial_t I(\mathbf{x}',t) \right\|^2 \mathrm{d}\mathbf{x}' \,.$$

Assume that the weighting function G is chosen such that $G(\mathbf{x}-\mathbf{x}') = 0$ for any $\mathbf{x}' \notin W(\mathbf{x})$.
In the following, we note $I_t = \partial_t I$ and $(I_{x_1}, I_{x_2})^\top = \nabla I$.

1. Prove that the minimizer $\mathbf{b}$ of $E(\mathbf{v})$ can be written as

   $$\mathbf{b} = -M^{-1}\mathbf{q}$$

   where the entries of M and $\mathbf{q}$ are given by

   $$m_{ij} = G * \left(I_{x_i} \cdot I_{x_j}\right) \quad \text{and} \quad q_i = G * (I_{x_i} \cdot I_t)$$

2. Show that if the gradient direction is constant in $W(\mathbf{x})$, i.e. $\nabla I(\mathbf{x}',t) = \alpha(\mathbf{x}',t)\mathbf{u}$ for a scalar function α and a 2D vector $\mathbf{u}$, M is not invertible.

   Explain how this observation is related to the aperture problem.

   Note: In the formalism of Lucas and Kanade, one cannot always estimate a translational motion. This problem is often referred to as the aperture problem. It arises for example, if the region in the window $W(x)$ around the point x has entirely constant intensity (for example a white wall), because then $\delta I(x) = 0$ and $I_t(x) = 0$ for all points in the window.

3. Write down explicit expressions for the two components b_1 and b_2 of the minimizer in terms of m_{ij} and q_i.

Note: $G * A$ denotes the convolution of image A with a kernel $G : \mathbb{R}^2 \to \mathbb{R}$ and is defined as

$$G * A = \int_{\mathbb{R}^2} G(\mathbf{x}-\mathbf{x}')A(\mathbf{x}')\mathrm{d}\mathbf{x}' \,.$$