Multiple View Geometry: Exercise 5

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Note: For more readability we will call $T_\times$ from the lecture $\hat{T}$ in this exercise sheet.

1. In this task, we are considering the non-zero essential matrix $E = \hat{T}R$ with $T \in \mathbb{R}^3$ and $R \in \mathrm{SO}(3)$. Let $R_Z\left(\pm\frac{\pi}{2}\right)$ be the rotation by $\pm\frac{\pi}{2}$ around the z-axis.

   Extra Information: The non-zero essential matrix has the singular value decomposition $E = U\Sigma V^T$ with $\Sigma = \mathrm{diag}\{\sigma, \sigma, 0\}$ for some $\sigma > 0$ and $U, V \in \mathrm{SO}(3)$. There exist exactly two options for $(\hat{T}, R)$:

   $$\left(\hat{T}_1, R_1\right) = \left(UR_Z\left(+\frac{\pi}{2}\right)\Sigma\, U^\top, \quad UR_Z\left(+\frac{\pi}{2}\right)^\top V^\top\right) \tag{1}$$

   $$\left(\hat{T}_2, R_2\right) = \left(UR_Z\left(-\frac{\pi}{2}\right)\Sigma\, U^\top, \quad UR_Z\left(-\frac{\pi}{2}\right)^\top V^\top\right) \tag{2}$$

   Show that by using (1) and (2), the following properties hold:

   (a) $\hat{T}_1, \hat{T}_2 \in so(3)$ (i.e. $\hat{T}_1, \hat{T}_2$ are skew-symmetric matrices)

   (b) $R_1, R_2 \in \mathrm{SO}(3)$ (i.e. R_1, R_2 are rotation matrices)

2. Consider the matrices $E = \hat{T}R$ and $H = R + Tu^\top$ with $R \in \mathbb{R}^{3\times 3}$ and $T, u \in \mathbb{R}^3$.
   Show that the following holds:

   (a) $E = \hat{T}H$

   (b) $H^\top E + E^\top H = 0$

3. Let $F \in \mathbb{R}^{3\times 3}$ be the fundamental matrix for the cameras C_1 and C_2. Show that the following holds for the epipoles e_1 and e_2:

   $$Fe_1 = 0 \qquad \text{and} \qquad e_2^\top F = 0$$

   Hint: Use the visualizations and information from the lecture (Chapter 6, Slides: 15/16) to determine e_1 and e_2.