Note: For more readability we will call $T_x$ from the lecture $\hat{T}$ in this exercise sheet.

1. In this task, we are considering the non-zero essential matrix $E = \hat{T}R$ with $T \in \mathbb{R}^3$ and $R \in SO(3)$. Let $R_Z\left(\pm\frac{\pi}{2}\right)$ be the rotation by $\pm\frac{\pi}{2}$ around the $z$-axis.

Extra Information: The non-zero essential matrix has the singular value decomposition $E = UV^T$ with $\Sigma = \text{diag}\{\sigma, \sigma, 0\}$ for some $\sigma > 0$ and $U, V \in SO(3)$. There exist exactly two options for $(\hat{T}, R)$:

$$
\begin{align*}
(\hat{T}_1, R_1) &= \left(UR_Z\left(\frac{\pi}{2}\right)\Sigma U^T, UR_Z\left(\frac{\pi}{2}\right)V^T\right) \quad \text{(1)} \\
(\hat{T}_2, R_2) &= \left(UR_Z\left(-\frac{\pi}{2}\right)\Sigma U^T, UR_Z\left(-\frac{\pi}{2}\right)V^T\right) \quad \text{(2)}
\end{align*}
$$

Show that by using (1) and (2), the following properties hold:

(a) $\hat{T}_1, \hat{T}_2 \in \text{so}(3)$ (i.e. $\hat{T}_1, \hat{T}_2$ are skew-symmetric matrices)

(b) $R_1, R_2 \in SO(3)$ (i.e. $R_1, R_2$ are rotation matrices)

2. Consider the matrices $E = \hat{T}R$ and $H = R + Tu^T$ with $R \in \mathbb{R}^{3 \times 3}$ and $T, u \in \mathbb{R}^3$. Show that the following holds:

(a) $E = \hat{T}H$

(b) $H^\top E + E^\top H = 0$

3. Let $F \in \mathbb{R}^{3 \times 3}$ be the fundamental matrix for the cameras $C_1$ and $C_2$. Show that the following holds for the epipoles $e_1$ and $e_2$:

$$Fe_1 = 0 \quad \text{and} \quad e_2^\top F = 0$$

Hint: Use the visualizations and information from the lecture (Chapter 6, Slides: 15/16) to determine $e_1$ and $e_2$. 

Exercise: June 14, 2023