# Multiple View Geometry: Solution 3

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**Image Formation**

1. Compute $\lambda$ and show that (2) is equivalent to

$$u = \frac{fX}{Z} + o_x\,, \quad v = \frac{fY}{Z} + o_y\,.$$

Performing the matrix multiplication in (2), one obtains

$$\begin{pmatrix}\lambda u\\ \lambda v\\ \lambda\end{pmatrix} = \begin{pmatrix} fX + o_x Z\\ fY + o_y Z\\ Z\end{pmatrix}$$

From the third row, it directly follows that $\lambda = Z$. Substituting $Z$ for $\lambda$ and dividing the equation by $Z$, one immediately obtains the result.

2. A classic ambiguity of the perspective projection is that one cannot tell an object from another object that is exactly *twice as big but twice as far*. Explain why this is true.
Let $\tilde{\mathbf{X}}_1 = (X_1\ Y_1\ Z_1)^\top$ be a point on the smaller object and $\tilde{\mathbf{X}}_2 = (X_2\ Y_2\ Z_2)^\top$ a point on the larger object. Since $\tilde{\mathbf{X}}_2$ is twice as far away, we have $Z_2 = 2Z_1$, and since it is twice as big we have $X_2 = 2X_1$ and $Y_2 = 2Y_1$. Thus,

$$u_2 = \frac{fX_2}{Z_2} + o_x = \frac{2fX_1}{2Z_1} + o_x = \frac{fX_1}{Z_1} + o_x = u_1$$

and analogous for $v_2 = v_1$.

3. For a camera with $f = 540$, $o_x = 320$ and $o_y = 240$, compute the pixel coordinates $u$ and $v$ of a point $\tilde{\mathbf{X}} = (60\ 100\ 180)^\top$.

$$u = \frac{fX}{Z} + o_x = \frac{540 \cdot 60}{180} + 320 = 500$$
$$v = \frac{fY}{Z} + o_y = \frac{540 \cdot 100}{180} + 240 = 540$$

Explain with the help of (b) why the units of $\tilde{\mathbf{X}}$ are not needed for this task.
Using different units (mm, cm, m, etc.) can be interpreted as scaling the point coordinates by a constant factor (10, 100, ...). The argument of (b) for a factor of 2 can easily be generalized to any factor $\alpha$.
Will the projected point be in the image if it has dimensions $640 \times 480$?
No, the point $(u, v) = (500, 540)$ is not in $[-0.5, 639.5] \times [-0.5, 479.5]$.

4. Using the generic projection $\pi$, show that (3) — and therefore also (1) and (2) — is equivalent to

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K \begin{pmatrix} \pi(\tilde{\mathbf{X}}) \\ 1 \end{pmatrix} .$$

Insert in the RHS of the equation:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K \begin{pmatrix} \pi(\tilde{\mathbf{X}}) \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & o_x \\ 0 & f & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X/Z \\ Y/Z \\ 1 \end{pmatrix} = \begin{pmatrix} fX/Z + o_x \\ fY/Z + o_y \\ 1 \end{pmatrix}$$