

Multiple View Geometry: Solution 7

Dr. Haoang Li, Daniil Sinitsyn, Sergei Solonets, Viktoria Ehm Computer Vision Group, TU Munich Wednesdays 16:00-18:15 at Hörsaal 2, "Interims I" (5620.01.102), and on RBG Live

Exercise: July 12, 2023

1. Robust Least Squares

- (a) What situations can you think of where a robust loss function might be needed? Missing (i.e. black or white) pixels in the image, dynamic changes of the scene, non-Lambertian surfaces like shiny or transparent objects, (local) changes in lighting conditions, ...
- (b) Write down the weight function for the Huber loss.

$$w_{\delta}(t) = egin{cases} 1 & |t| \leq \delta \ rac{\delta}{|t|} & ext{else} \end{cases}$$

2. Optimization Techniques

Write down the update step $\Delta \xi$ for each of the following minimization methods:

(a) Gradient descent, normal least squares,

$$\Delta \xi = -\lambda J^{\top} \mathbf{r}$$

(b) Gradient descent, weighted least squares,

$$\Delta \xi = -\lambda J^{\top} W \mathbf{r}$$

(c) Gauss-Newton, normal least squares,

$$\Delta \boldsymbol{\xi} = -(\boldsymbol{J}^{\top}\boldsymbol{J})^{-1}\boldsymbol{J}^{\top}\mathbf{r}$$

(d) Gauss-Newton, weighted least squares,

$$\Delta \xi = -(J^{\top}WJ)^{-1}J^{\top}W\mathbf{r}$$

For weighted least square, we can use a change of variable to write the optimization problem exactly as a normal least square :

$$\sum_{i} w(r_i) r_i(\xi)^2 = \sum_{i} (\sqrt{w(r_i)} r_i(\xi))^2 = \sum_{i} \tilde{r}_i(\xi)^2 = \|\tilde{r}(\xi)\|^2$$

Then to get the new update steps, we just substitute r with $\tilde{r} = \sqrt{W}r$ and J with $\tilde{J} = \sqrt{W}J$ in the previous one.

 $(\tilde{J} \text{ is the jacobian of } \tilde{r}, \text{ and } \sqrt{W} \text{ is the matrix whose components are the square root of the components of } W).}$