Multiple View Geometry: Exercise Sheet 2

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Part I: Theory

1. Given a rotation matrix $R \in SO(3)$, show that it is indeed a rigid body motion.

   Hint: Consider rotating a vector v with R, what properties should Rv satisfy so that R is a rigid body motion?

2. Let A be a real symmetric matrix, and λ_a, λ_b eigenvalues with eigenvectors v_a and v_b. Prove: if v_a and v_b are not orthogonal, it follows: $\lambda_a = \lambda_b$.

   Hint: What can you say about $\langle Av_a, v_b \rangle$?

3. Given two unit vectors u and v (i.e. $\|u\| = \|v\| = 1$), show that the vector $w := u + v$ bisect the angle between u and v.

   Hint: Denote the angle between u and v as θ and the angle between w and u as α, what can you say about $\langle u, v \rangle$ and $\langle u, w \rangle$?

4. Let $A \in \mathbb{R}^{n \times n}$ be a symmetric matrix with the orthonormal basis of eigenvectors $v_1, \ldots, v_n$ and eigenvalues $\lambda_1 \geq \ldots \geq \lambda_n$. Find all vectors x, that minimize the following term:

   $$\min_{||x||=1} x^\top A x$$

   How many solutions exist? How can the term be maximized?

   Hint: Use the expression $x = \sum_{i=1}^{n} \alpha_i v_i$ with coefficients $\alpha_i \in \mathbb{R}$ and compute appropriate coefficients!

5. Let $A \in \mathbb{R}^{m \times n}$. Prove that $\text{kernel}(A) = \text{kernel}(A^\top A)$.

   Hint: Consider a) $x \in \text{kernel}(A) \Rightarrow x \in \text{kernel}(A^\top A)$
   and b) $x \in \text{kernel}(A^\top A) \Rightarrow x \in \text{kernel}(A)$.

6. Singular Value Decomposition (SVD)

   Let $A = USV^\top$ be the SVD of A.

   (a) Write down possible dimensions for A, U, S and V.

   (b) What are the similarities and differences between the SVD and the eigenvalue decomposition?

(c) What do you know about the relationship between U, S, V and the eigenvalues and eigenvectors of $A^\top A$ and $AA^\top$?

(d) What is the interpretation of the entries in S and what do the entries of S tell us about A?