Multiple View Geometry: Exercise Sheet 9

Prof. Dr. Daniel Cremers,
Mohammed Brahimi, Zhenzhang Ye, Regine Hartwig
Computer Vision Group, TU Munich
`Wednesdays 16:15-17:45 at Hörsaal 2, "Interims I" (5620.01.102), and on RBG Live`

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Part I: Theory

This exercise partly builds upon the theory part of last week's exercise. Check the solutions of Sheet 8 in case there is something you do not understand.

1. **Robust Least Squares**

   In order to make the solution of the Direct Image Alignment from Sheet 8 more robust to outliers, one can replace the square in the energy

   $$E(\xi) = \sum_i r_i(\xi)^2$$

   by a robust loss function ρ:

   $$E_\rho(\xi) = \sum_i \rho\left(r_i(\xi)\right).$$

   (a) What situations can you think of where a robust loss function might be needed?

   The minimizer of E_ρ also minimizes the weighted least squares problem

   $$E_w(\xi) = \sum_i w(r_i) r_i(\xi)^2$$

   with weights defined by $w(t) := \rho'(t)/t$.

   (b) One example for a robust loss function is the Huber loss function ρ_δ:

   $$\rho_\delta(t) = \begin{cases} \frac{t^2}{2} & |t| \leq \delta \\ \delta|t| - \frac{\delta^2}{2} & \text{else} \end{cases}$$

   Write down the weight function for the Huber loss.

2. **Optimization Techniques**

   Define $\mathbf{r}$ as the vector containing the residuals and J as the matrix containing gradients of all residuals at $\xi = \mathbf{0}$:

$$\mathbf{r}_i = r_i(\mathbf{0})\,, \quad J^{(i)} = \left.\frac{\partial r_i(\xi)}{\partial \xi}\right|_{\xi=\mathbf{0}} .$$

   Furthermore, let W be the diagonal matrix with weights $w(r_i(\mathbf{0}))$ on the diagonal. Write down the update step $\Delta\xi$ for each of the following minimization methods:

   (a) Gradient descent, normal least squares,

   (b) Gradient descent, weighted least squares,

   (c) Gauss-Newton, normal least squares,

   (d) Gauss-Newton, weighted least squares,

   (e) Levenberg-Marquardt, normal least squares, and

   (f) Levenberg-Marquardt, weighted least squares.