Practical Course: Vision Based Navigation

Lecture 4: Structure from Motion (SfM)

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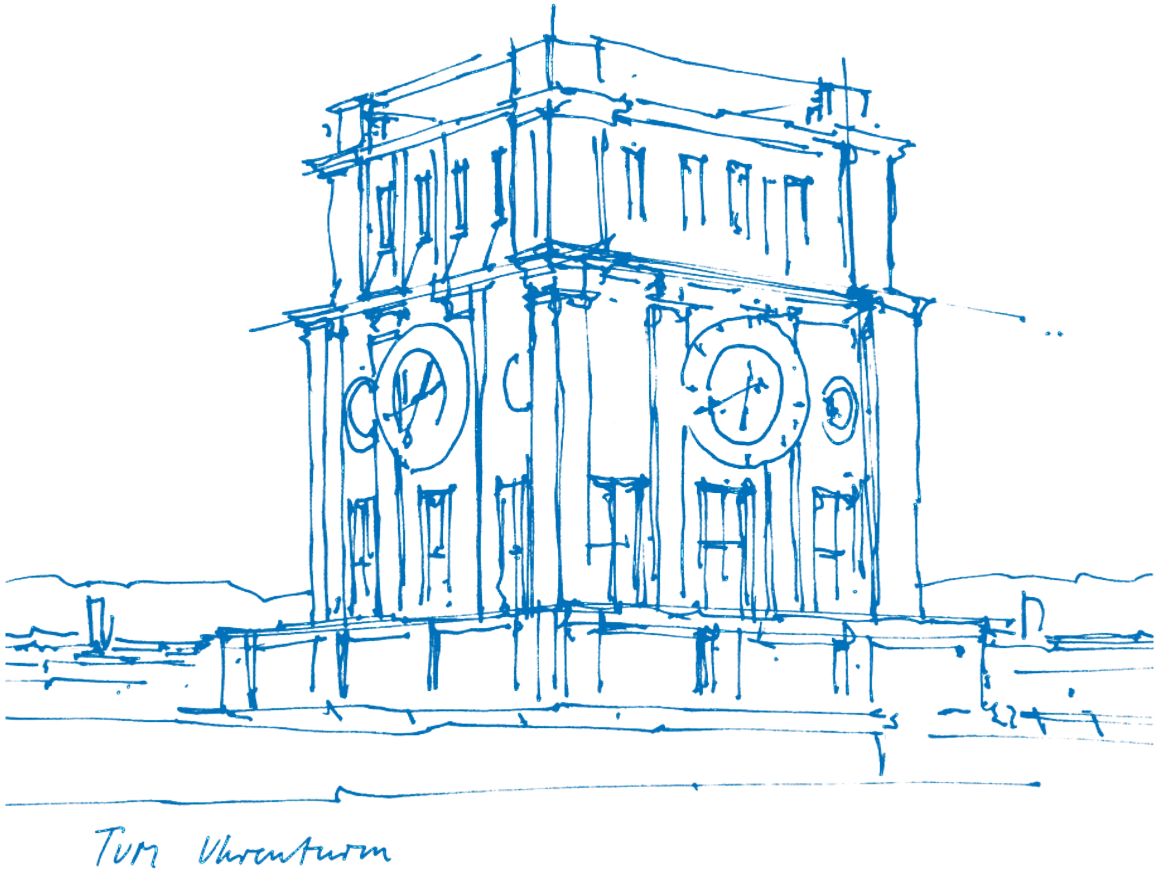

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Topics Covered

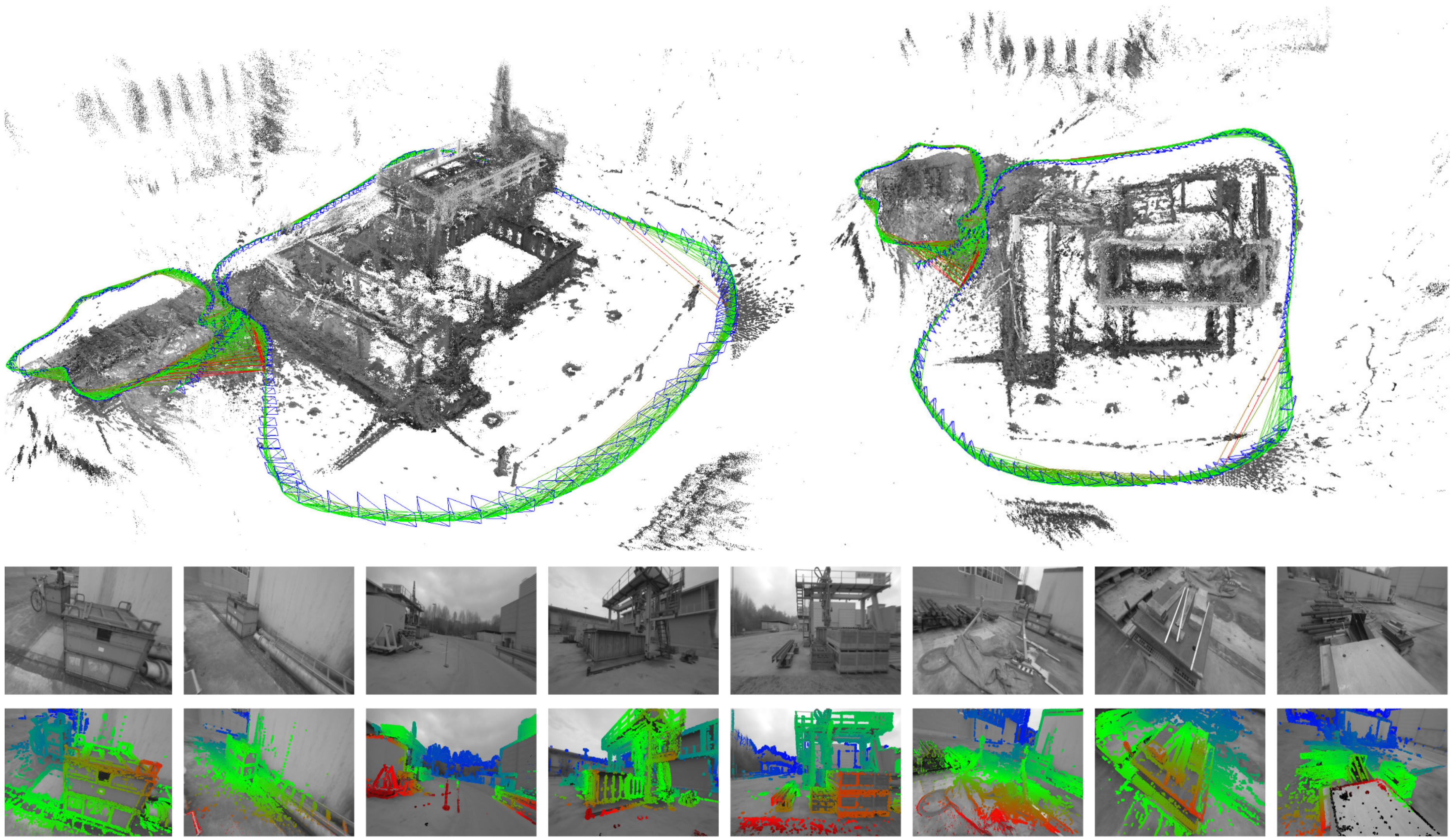
- Introduction
  - Structure from Motion (SfM)
  - Simultaneous Localization and Mapping (SLAM)
- Bundle Adjustment
  - Energy Function
  - Non-linear Least Squares
  - Exploiting the Sparse Structure
- Triangulation

Structure from Motion

Agarwal et al., “Building Rome in a day”, ICCV 2009, “Dubrovnik” image set

- 3D reconstruction using a set of unordered images
- Requires estimation of 6DoF poses

Simultaneous Localization and Mapping (SLAM)



Engel et al., “LSD-SLAM: Large-Scale Direct Monocular SLAM”, ECCV 2014

- Estimate 6DoF poses and map from sequential image data
- Update once new frames arrive

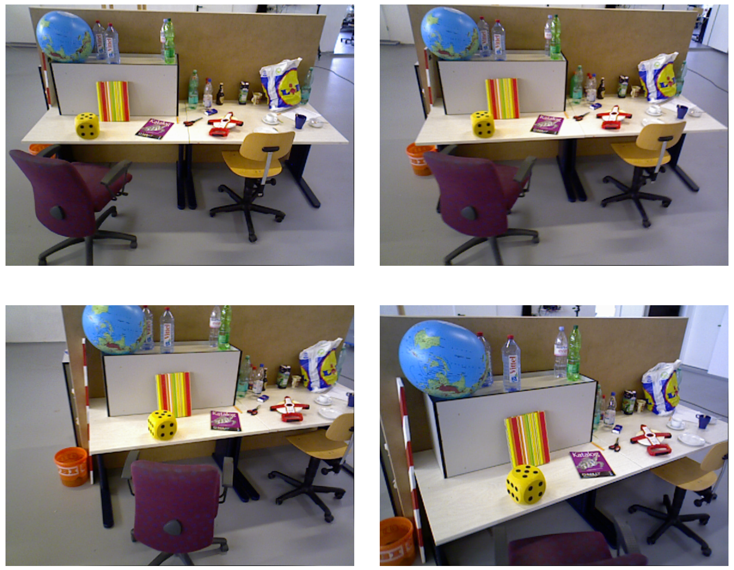
Problem Definition SfM / Visual SLAM

Estimate camera poses and map from a set of images

- Input

  Set of images $I_{0:t} = \{I_0, I_1, \ldots, I_t\}$

  Additional input possible
  - Stereo
  - Depth
  - Inertial measurements
  - Control input



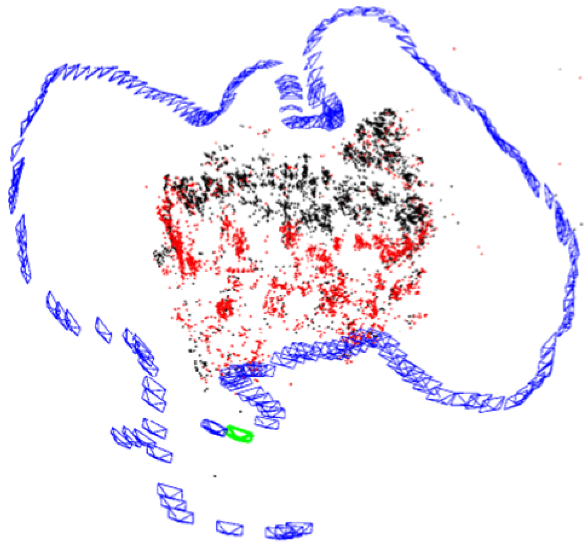
fr3/long_office_household sequence,
TUM RGB-D benchmark

- Output

  Camera pose estimates $\mathbf{T}_i \in \mathrm{SE}(3)$,

  also written as $\boldsymbol{\xi}_i = \left(\log \mathbf{T}_i\right)^\vee$ $\quad i \in \{0,1,\ldots,t\}$

  Environment map M



Mur-Artal et al., 2015

Typical SfM Pipeline

1) **Map initialization**
   - Using 2D-to-2D correspondences
   - Recover pose (stereo pair if available)
   - Triangulate landmarks using pose

2) **Localization** with known map
   - Using 2D-to-3D correspondences

3) **Mapping** with known poses
   - Using 2D-to-2D correspondences
     → Triangulation

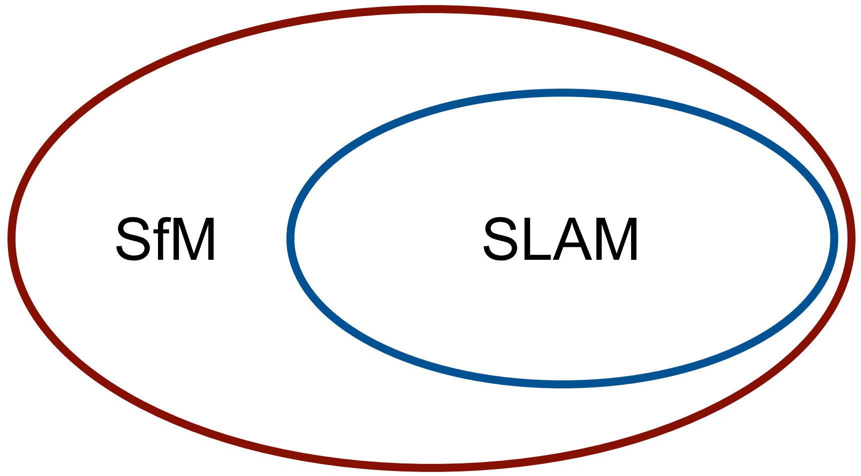
4) **Joint refinement** of map and poses
   - Using 2D-to-2D correspondences
     → Bundle adjustment

Iterate to add new frames

Visual SLAM

SLAM ⊂ SfM, with special focus:

- Sequential image data
- Data arrives sequentially
- Preferably realtime
- More focus on trajectory



Technical solutions:

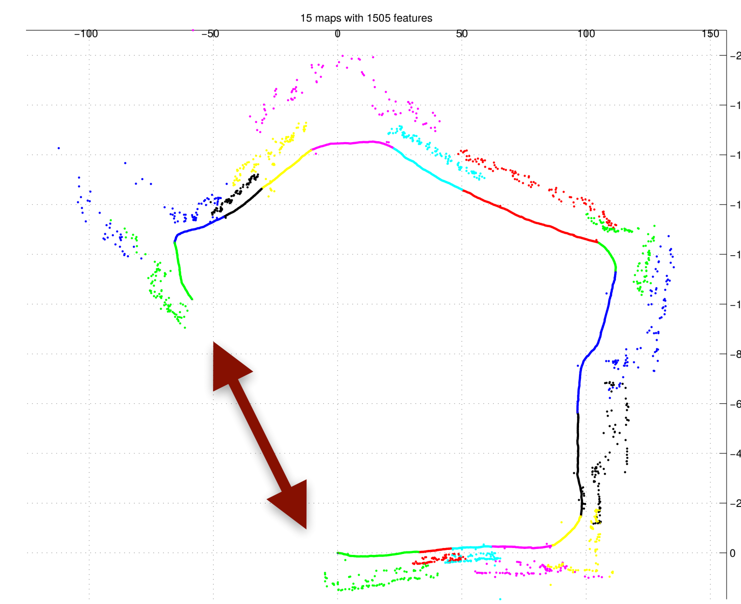
- Windowed optimization
- Selection of keyframes
- Removal of keyframes (e.g. marginalization)

→ Accumulation of drift

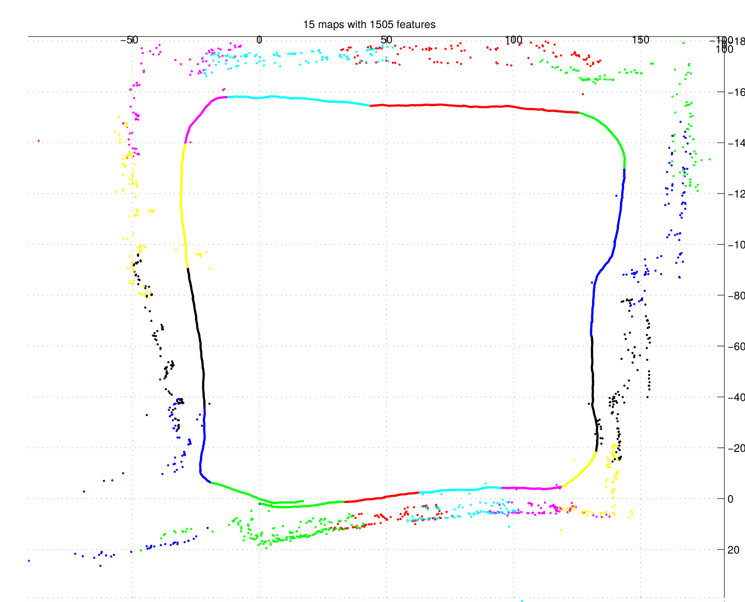
- Detect loop closures
- Global mapping in separate thread (e.g. pose graph optimization)

Odometry

- No global mapping
- Incremental tracking only
- Local map possible

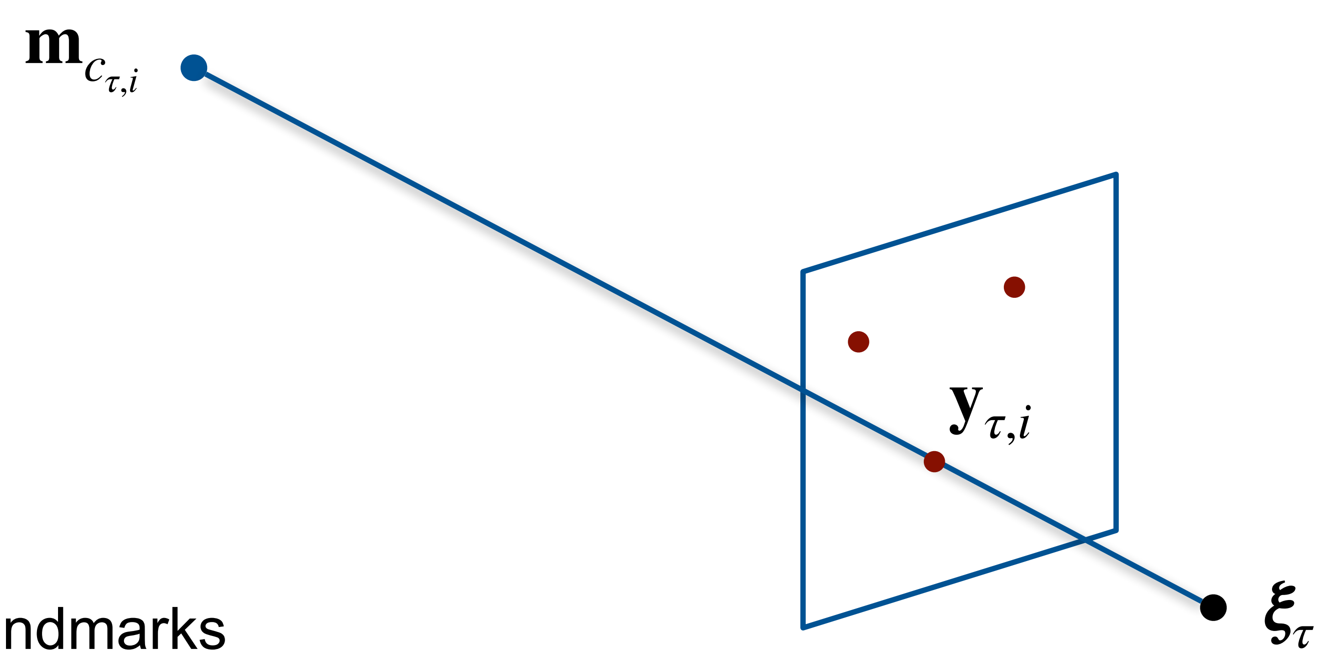


Loop closure



Clemente et al., RSS 2007

Landmarks and Features



- The map consists of 3D locations of landmarks

$$M = \{\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_S\}$$

- For image τ, the set of 2D image coordinates of detected features is denoted

$$Y_\tau = \{\mathbf{y}_{\tau,1}, \mathbf{y}_{\tau,2}, \ldots, \mathbf{y}_{\tau,N}\}$$

- Known data association:
  Feature i in image τ corresponds to landmark $j = c_{\tau,i}$ $\qquad (1 \leq i \leq N, 1 \leq j \leq S)$

Bundle Adjustment Energy

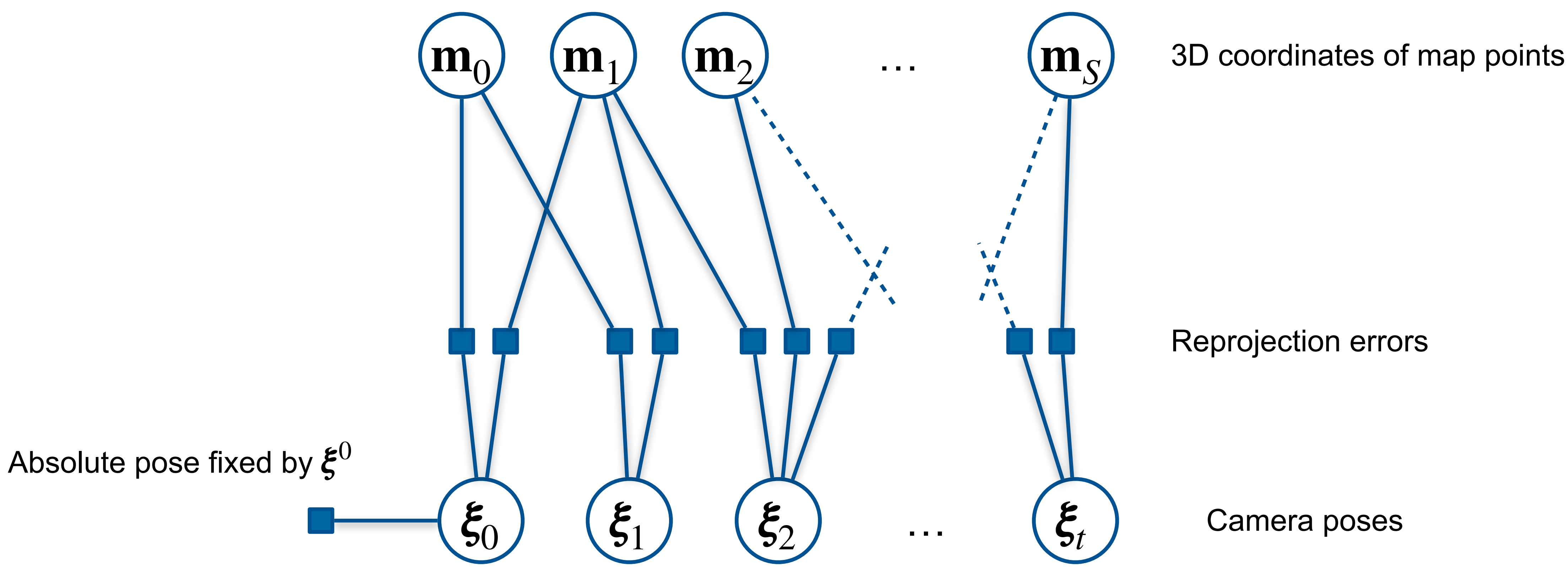
$$E\left(\boldsymbol{\xi}_{0:t}, M\right) = \frac{1}{2}\left(\boldsymbol{\xi}_0 \ominus \boldsymbol{\xi}^0\right)^\top \boldsymbol{\Sigma}_{0,\xi}^{-1}\left(\boldsymbol{\xi}_0 \ominus \boldsymbol{\xi}^0\right)$$

Absolute pose prior

$$+\frac{1}{2}\sum_{\tau=0}^{t}\sum_{i=1}^{N_\tau}\left(\mathbf{y}_{\tau,i} - h\left(\boldsymbol{\xi}_\tau, \mathbf{m}_{c_{\tau,i}}\right)\right)^\top \boldsymbol{\Sigma}_{\mathbf{y}_{\tau,i}}^{-1}\left(\mathbf{y}_{\tau,i} - h\left(\boldsymbol{\xi}_\tau, \mathbf{m}_{c_{\tau,i}}\right)\right)$$

Reprojection error

- Pose prior: Fix absolute pose ambiguity
  - In this case equivalent to keeping $\boldsymbol{\xi}_0 = \boldsymbol{\xi}^0$
  - Keep absolute pose information e.g. when first frame is marginalized
- Additional prior to fix scale ambiguity might be necessary



Energy Function as Non-linear Least Squares

- Residuals as function of state vector $\mathbf{x}$

$$\mathbf{r}^0(\mathbf{x}) := \boldsymbol{\xi}_0 \ominus \boldsymbol{\xi}^0$$

$$\mathbf{r}^{y}_{t,i}(\mathbf{x}) := \mathbf{y}_{t,i} - h\left(\boldsymbol{\xi}_t, \mathbf{m}_{c_{t,i}}\right)$$

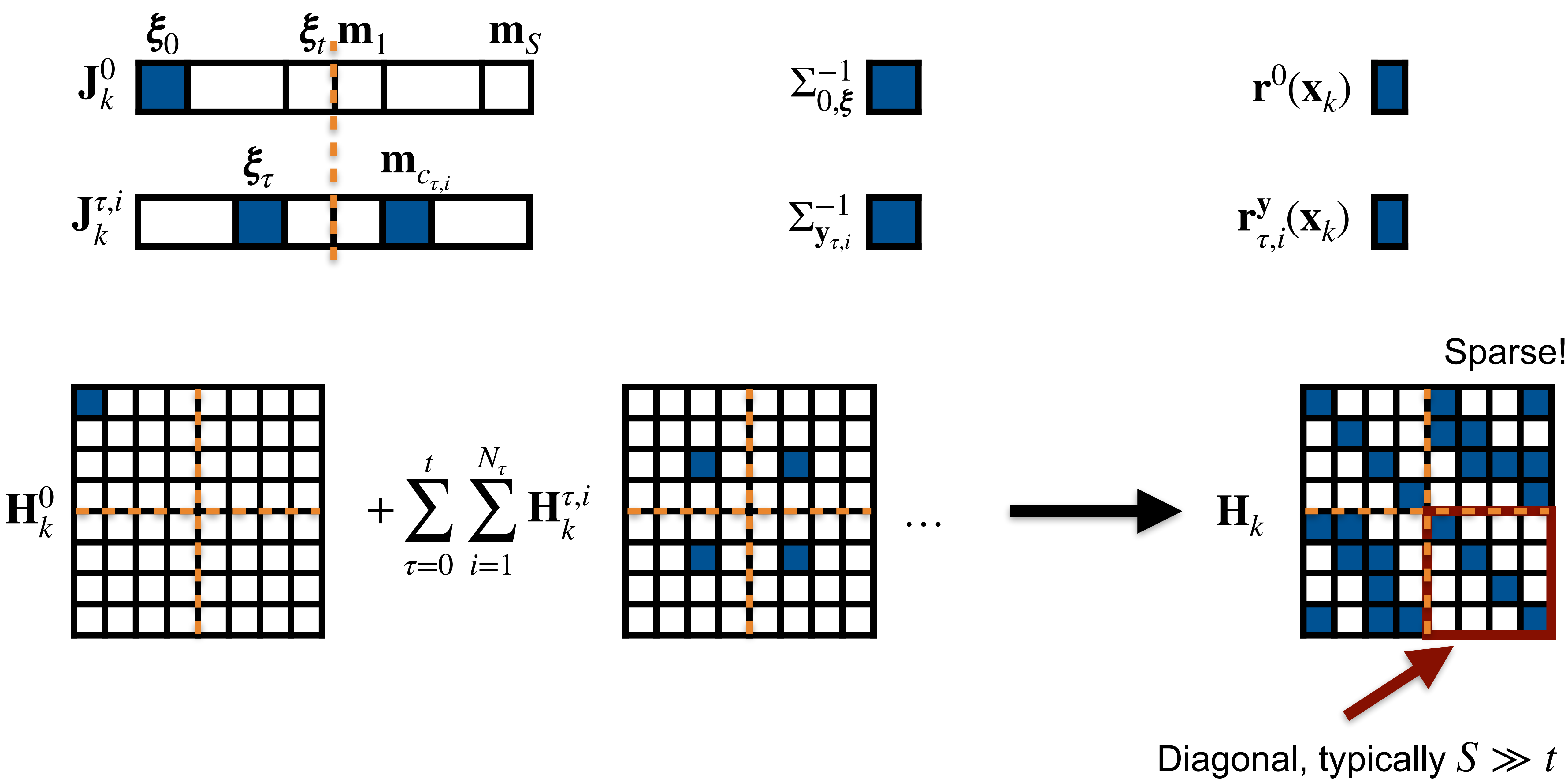
$$\mathbf{x} := \begin{pmatrix} \boldsymbol{\xi}_0 \\ \vdots \\ \boldsymbol{\xi}_t \\ \mathbf{m}_1 \\ \vdots \\ \mathbf{m}_S \end{pmatrix}$$

- Stack the residuals in a vector-valued function und collect the residual covariances on the diagonal blocks of a square matrix

$$\mathbf{r}(\mathbf{x}) := \begin{pmatrix} \mathbf{r}^0(\mathbf{x}) \\ \mathbf{r}^{\mathbf{y}}_{0,1}(\mathbf{x}) \\ \vdots \\ \mathbf{r}^{\mathbf{y}}_{t,N_t}(\mathbf{x}) \end{pmatrix} \qquad \mathbf{W} := \begin{pmatrix} \boldsymbol{\Sigma}^{-1}_{0,\xi} & 0 & \cdots & 0 \\ 0 & \boldsymbol{\Sigma}^{-1}_{\mathbf{y}_{0,1}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \boldsymbol{\Sigma}^{-1}_{\mathbf{y}_{t,N_t}} \end{pmatrix}$$

- Rewrite energy function as $\quad E(\mathbf{x}) = \dfrac{1}{2}\mathbf{r}(\mathbf{x})^\top \mathbf{W}\mathbf{r}(\mathbf{x})$

Structure of the Bundle Adjustment Problem

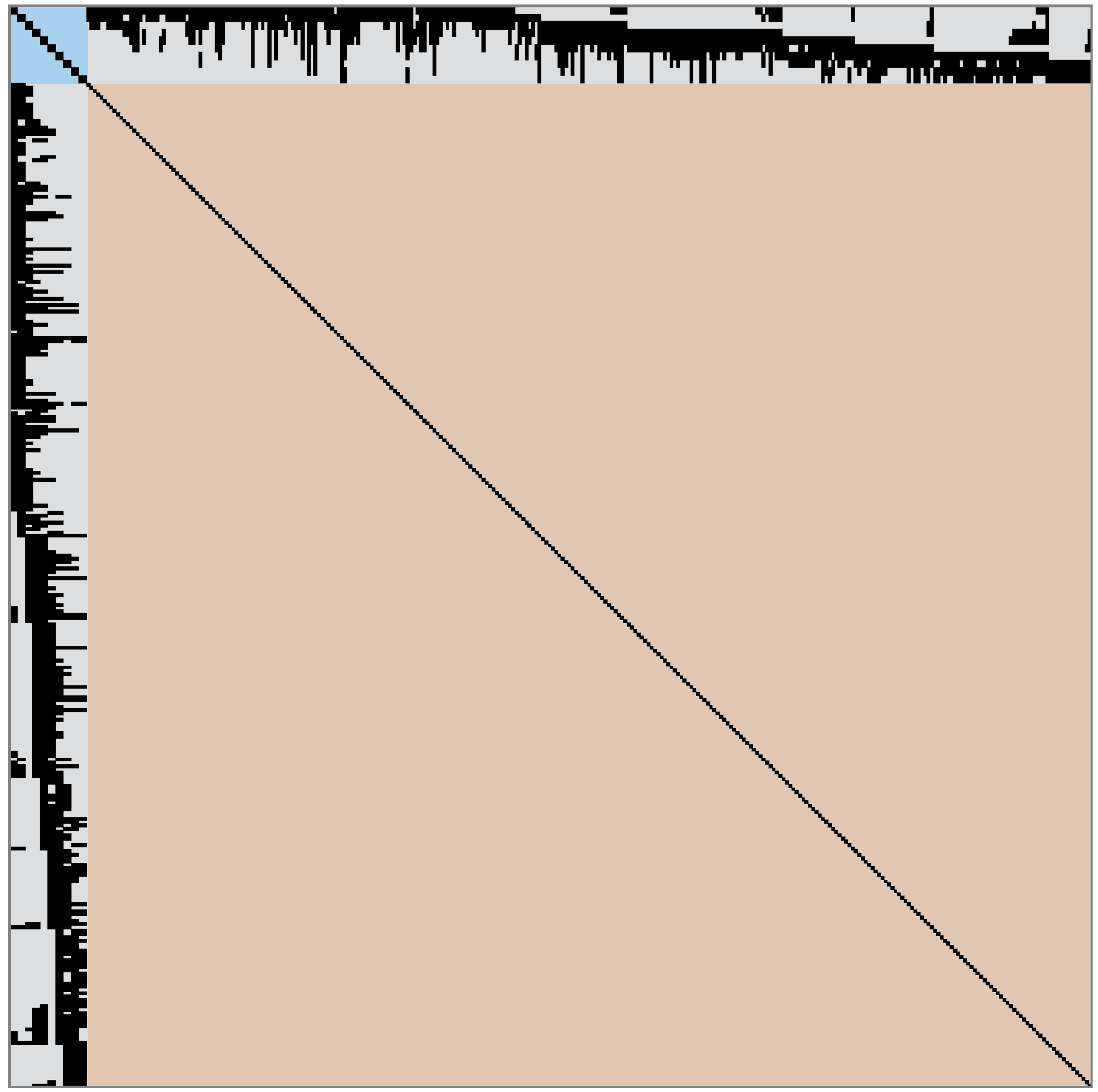


$$\mathbf{H}_k = \mathbf{H}_k^0 + \sum_{\tau=0}^{t}\sum_{i=1}^{N_\tau}\mathbf{H}_k^{\tau,i} = \left(\mathbf{J}_k^0\right)^\top \boldsymbol{\Sigma}_{0,\xi}^{-1}\left(\mathbf{J}_k^0\right) + \sum_{\tau=0}^{t}\sum_{i=1}^{N_\tau}\left(\mathbf{J}_k^{\tau,i}\right)^\top \boldsymbol{\Sigma}_{\mathbf{y}_{\tau,i}}^{-1}\left(\mathbf{J}_k^{\tau,i}\right)$$

Example Hessian of a BA Problem

Pose dimensions
(10 poses)

$$\boldsymbol{H}_k =$$



Landmark dimensions
(982 landmarks)

Lourakis et al., 2009

Large, but sparse!

How to invert efficiently?

Exploiting the Sparse Structure

- Idea:
  Apply the Schur complement to solve the system in a partitioned way

$$\mathbf{H}_k \Delta \mathbf{x} = -\mathbf{b}_k \quad \longrightarrow \quad \begin{pmatrix} \mathbf{H}_{\xi\xi} & \mathbf{H}_{\xi\mathrm{m}} \\ \mathbf{H}_{\mathrm{m}\xi} & \mathbf{H}_{\mathrm{mm}} \end{pmatrix} \begin{pmatrix} \Delta \mathbf{x}_{\xi} \\ \Delta \mathbf{x}_{\mathrm{m}} \end{pmatrix} = -\begin{pmatrix} \mathbf{b}_{\xi} \\ \mathbf{b}_{\mathrm{m}} \end{pmatrix}$$

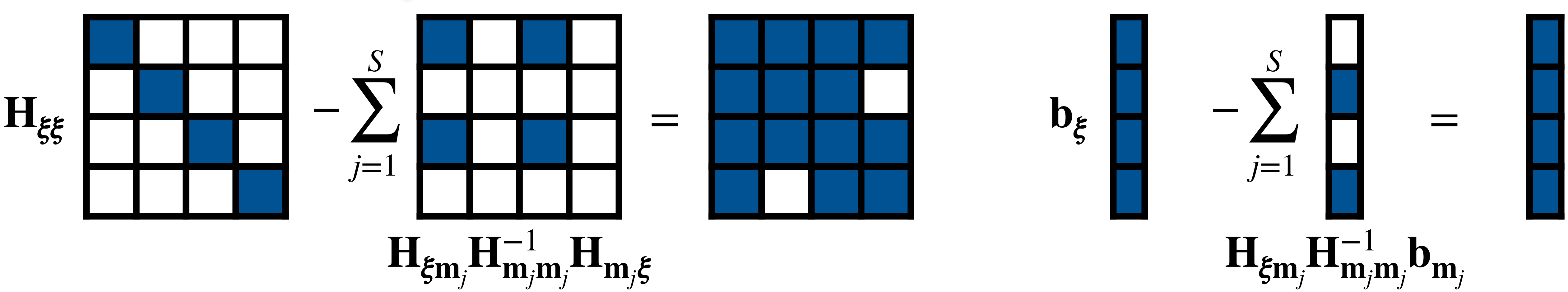
$$\longrightarrow \quad \Delta \mathbf{x}_{\xi} = -\left(\mathbf{H}_{\xi\xi} - \mathbf{H}_{\xi\mathrm{m}} \mathbf{H}_{\mathrm{mm}}^{-1} \mathbf{H}_{\mathrm{m}\xi} \right)^{-1} \left(\mathbf{b}_{\xi} - \mathbf{H}_{\xi\mathrm{m}} \mathbf{H}_{\mathrm{mm}}^{-1} \mathbf{b}_{\mathrm{m}} \right)$$

$$\longrightarrow \quad \Delta \mathbf{x}_{\mathrm{m}} = -\mathbf{H}_{\mathrm{mm}}^{-1} \left(\mathbf{b}_{\mathrm{m}} + \mathbf{H}_{\mathrm{m}\xi} \Delta \mathbf{x}_{\xi} \right)$$

- Is this any better?

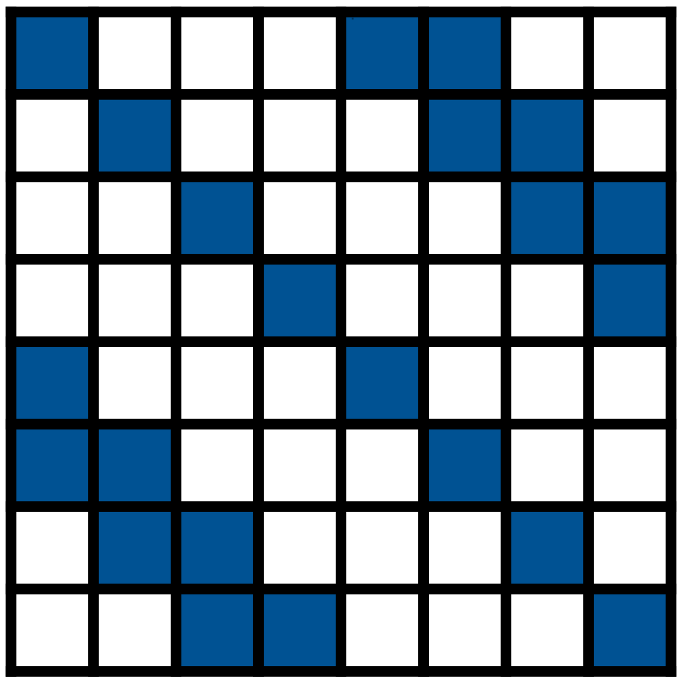
Exploiting the Sparse Structure

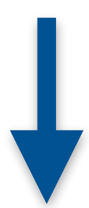
$$\Delta \mathbf{x}_\xi = -\left(\mathbf{H}_{\xi\xi} - \mathbf{H}_{\xi \mathbf{m}}\mathbf{H}_{\mathbf{mm}}^{-1}\mathbf{H}_{\mathbf{m}\xi}\right)^{-1}\left(\mathbf{b}_\xi - \mathbf{H}_{\xi \mathbf{m}}\mathbf{H}_{\mathbf{mm}}^{-1}\mathbf{b}_{\mathbf{m}}\right)$$



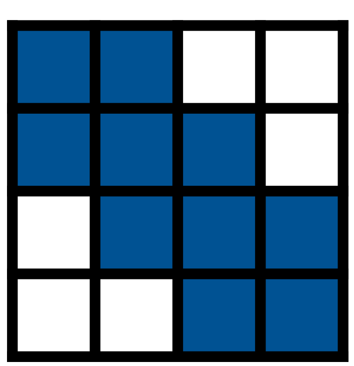
Effect of Loop Closures on the Hessian

Full Hessian

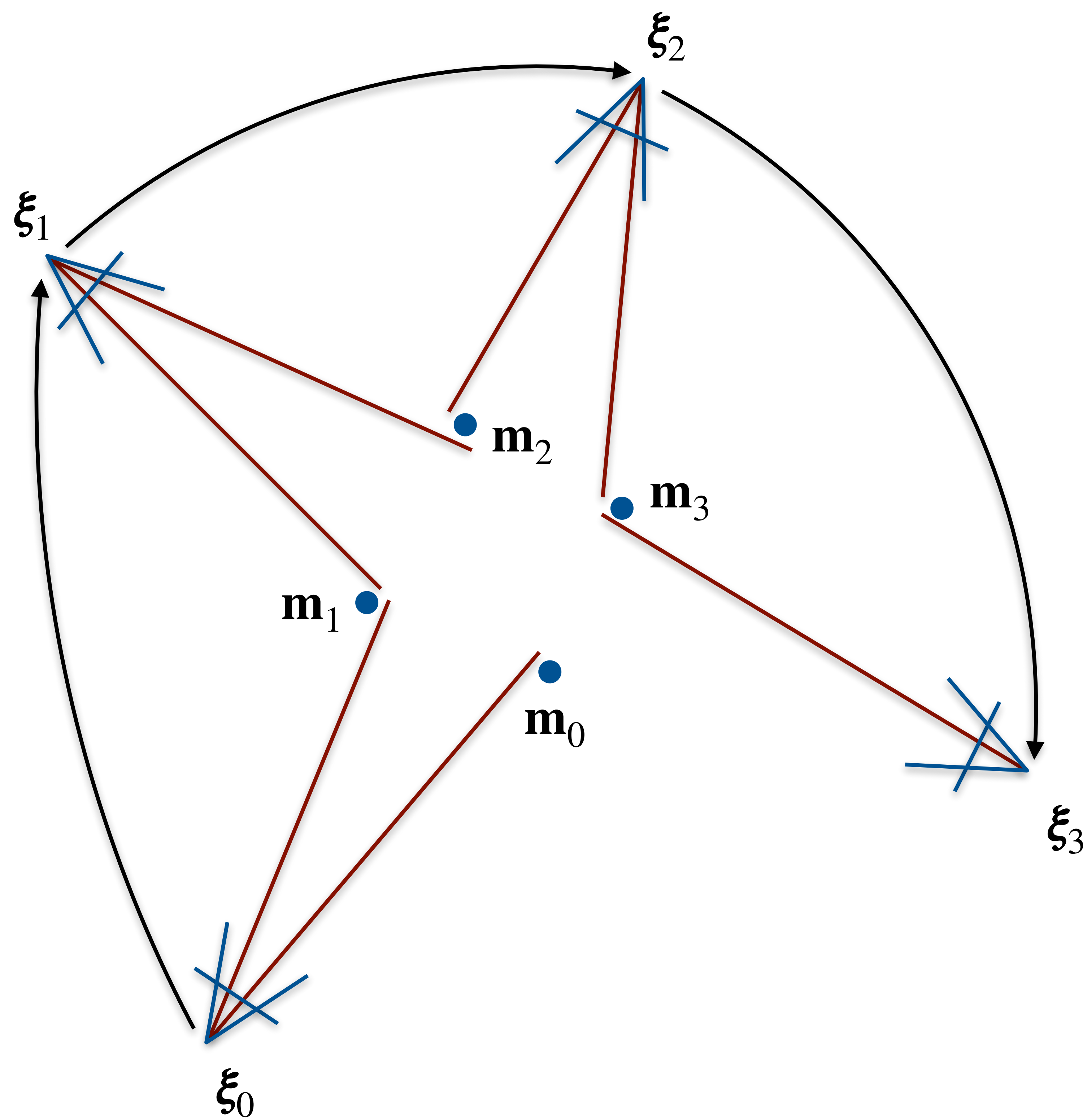




Reduced pose Hessian



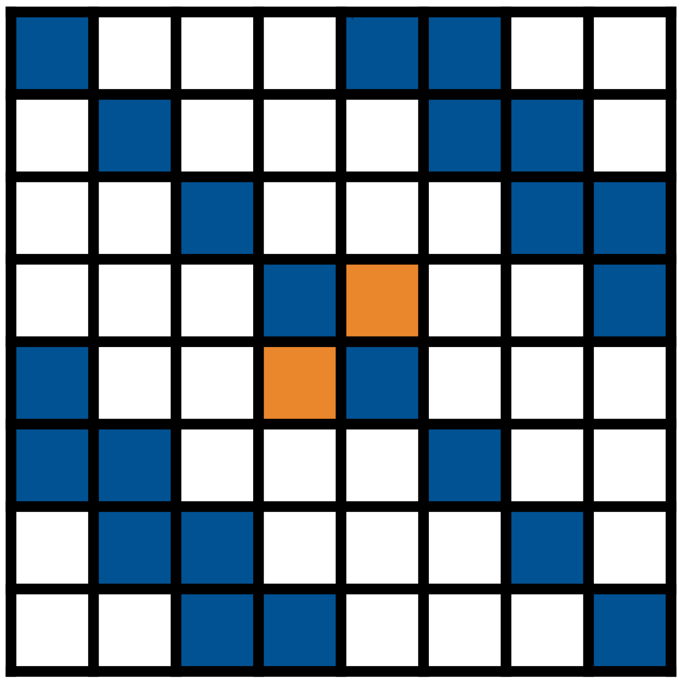
Band matrix



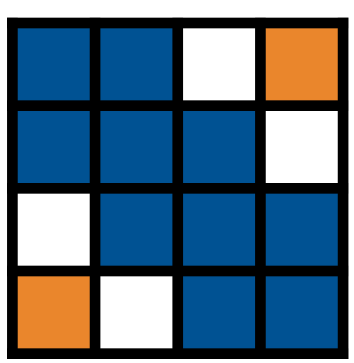
Before loop closure

Effect of Loop Closures on the Hessian

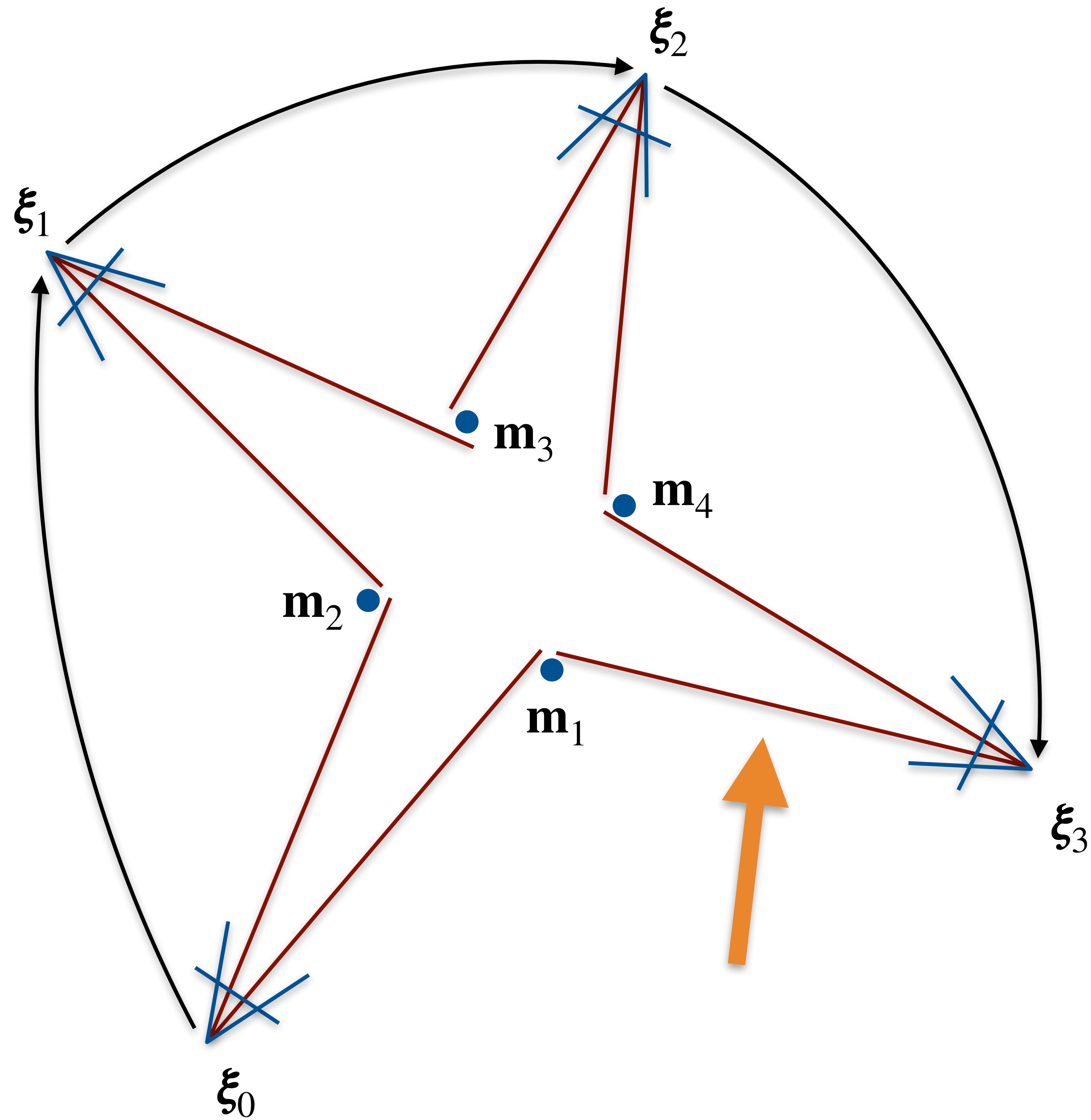
Full Hessian



Reduced pose Hessian



No band matrix: costlier to solve



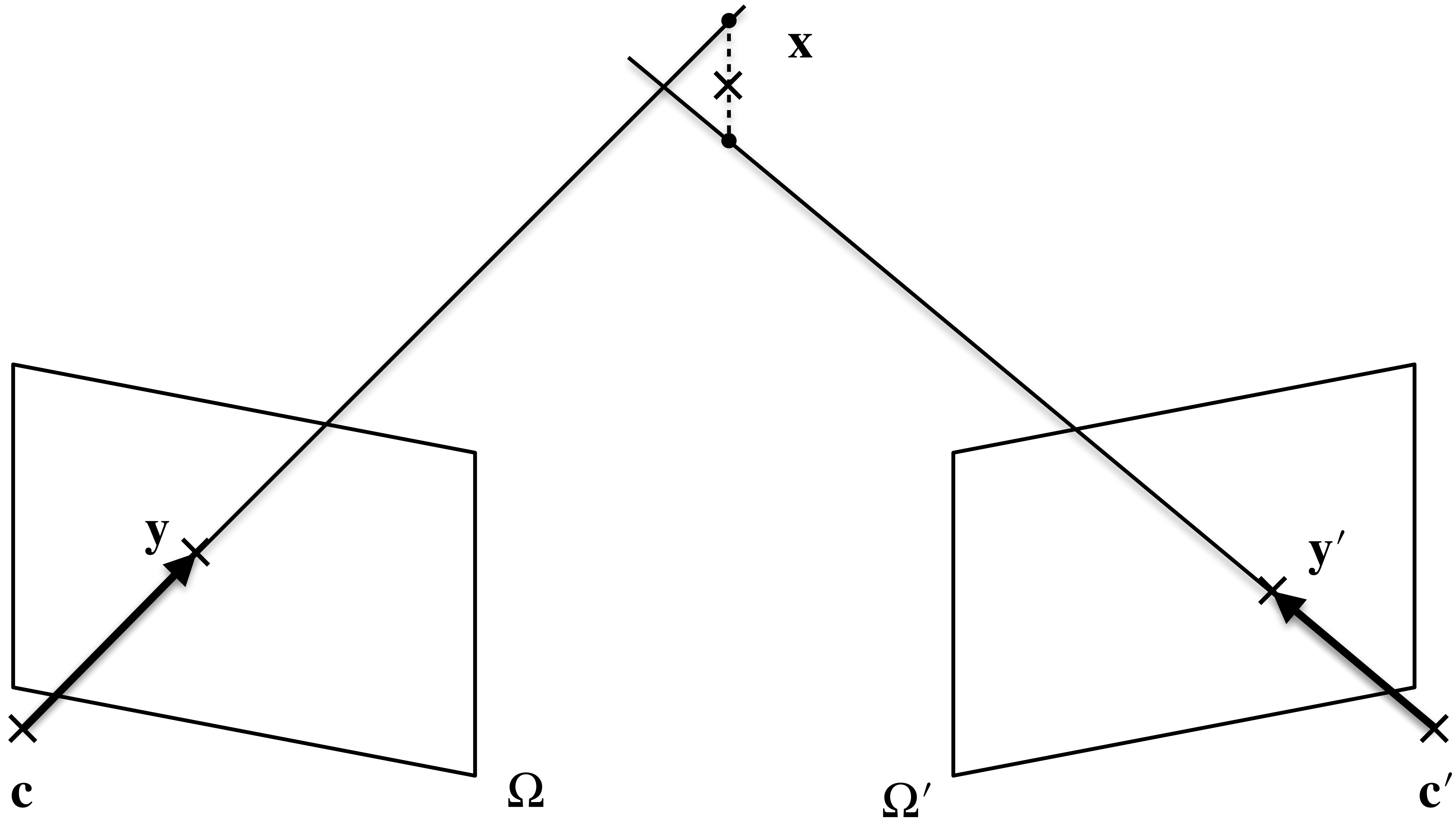
After loop closure

Further Considerations

Many methods to improve convergence / robustness / run-time efficiency, e.g.

- Use matrix decompositions (e.g. Cholesky) to perform inversions
- Levenberg-Marquardt optimization improves basin of convergence
- Heavier-tail distributions / robust norms on the residuals can be implemented using iteratively reweighted least squares
- Preconditioning
- Hierarchical optimization
- Variable reordering
- Delayed relinearization

Triangulation



- Find landmark position given the camera poses
- Ideally, the rays should intersect
- In practice, many sources of error: pose estimates, feature detections and camera model / intrinsic parameters

Triangulation

- Goal: Reconstruct 3D point $\tilde{\mathbf{x}} = (x, y, z, w)^\top \in \mathbb{P}^3$ from 2D image observations $\{\mathbf{y}_1, \ldots, \mathbf{y}_N\}$ for known camera poses $\{\mathbf{T}_1, \ldots, \mathbf{T}_N\}$

- Linear solution: Find 3D point such that reprojections equal its projection
  - For each image i, let $\mathbf{T}_i = \begin{pmatrix} & \mathbf{p}_1 & & \\ & \mathbf{p}_2 & & \\ & \mathbf{p}_3 & & \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and $\mathbf{y}_i = \begin{pmatrix} u \\ v \end{pmatrix}$
  - Projecting $\tilde{\mathbf{x}}$ yields $\mathbf{y}'_i = \pi\left(\mathbf{T}_i \tilde{\mathbf{x}}\right) = \begin{pmatrix} \mathbf{p}_1\tilde{\mathbf{x}}/\mathbf{p}_3\tilde{\mathbf{x}} \\ \mathbf{p}_2\tilde{\mathbf{x}}/\mathbf{p}_3\tilde{\mathbf{x}} \end{pmatrix}$
  - Requiring $\mathbf{y}'_i = \mathbf{y}_i$ gives two linear equations per image: $$\mathbf{p}_1\tilde{\mathbf{x}} = u\mathbf{p}_3\tilde{\mathbf{x}}$$ $$\mathbf{p}_2\tilde{\mathbf{x}} = v\mathbf{p}_3\tilde{\mathbf{x}}$$
  - Leads to system of linear equations $\mathbf{A}\tilde{\mathbf{x}} = \mathbf{0}$, two approaches to solve:
    - Set $w = 1$ and solve non-homogeneous least squares problem
    - Find nullspace of $\mathbf{A}$ using SVD, then scale such that $w = 1$

- Non-linear least squares on reprojection errors (more accurate): $$\min_{\mathbf{x}} \left\{ \sum_{i=1}^{N} \|\mathbf{y}_i - \mathbf{y}'_i\|_2^2 \right\}$$

- Different solutions for different methods in the presence of noise

Exercises

Exercise sheet 4

- Implement components of SfM pipeline
- BA: Ceres can do the Schur complement
- Triangulation: use OpenGV's triangulate function

```
ceres::Solver::Options ceres_options;
ceres_options.max_num_iterations = 20;
ceres_options.linear_solver_type =
ceres::SPARSE_SCHUR;
ceres_options.num_threads = 8;
ceres::Solver::Summary summary;
Solve(ceres_options, &problem,
&summary);
std::cout << summary.FullReport() <<
std::endl;
```

Next slide

Exercise sheet 5

- Implement components of odometry
- Similar to sheet 4, but:
  - More efficient 2D-3D matching using approximate pose of previous frame
  - New keyframe if number of matches too small
  - New landmarks by triangulation from stereo pair
  - Keep runtime bounded by removing old keyframes

Summary

SfM
- Estimate map and camera poses from set of images
- SLAM: Sequential data, real-time
- Odometry: No global mapping

Bundle Adjustment
- Non-linear least squares problem
- Sparse structure of Hessian can be exploited for efficient inversion

Triangulation
- Linear and non-linear algorithms
- Differences in the presence of noise