Multiple View Geometry: Exercise Sheet 5

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`Wednesdays 16:15-18:15 at Hörsaal 2, "Interims I" (5620.01.102), and on RBG Live`

Exercise: May 29th, 2025

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1. **The Lucas-Kanade method**

   The weighted Lucas-Kanade energy $E(\mathbf{v})$ is defined as

$$E(\mathbf{v}) = \int_{W(\mathbf{x})} G(\mathbf{x} - \mathbf{x}') \left\| \nabla I(\mathbf{x}', t)^\top \mathbf{v} + \partial_t I(\mathbf{x}', t) \right\|^2 \mathrm{d}\mathbf{x}' \,.$$

   Assume that the weighting function G is chosen such that $G(\mathbf{x} - \mathbf{x}') = 0$ for any $\mathbf{x}' \notin W(\mathbf{x})$. In the following, we note $I_t = \partial_t I$ and $(I_{x_1}, I_{x_2})^\top = \nabla I$.

   (a) Prove that the minimizer $\hat{\mathbf{v}}$ of $E(\mathbf{v})$ can be written as

$$\hat{\mathbf{v}} = -M^{-1}\mathbf{q}$$

   where the entries of M and $\mathbf{q}$ are given by

$$m_{ij} = G * \left(I_{x_i} \cdot I_{x_j}\right) \quad \text{and} \quad q_i = G * \left(I_{x_i} \cdot I_t\right)$$

   (b) Show that if the gradient direction is constant in $W(\mathbf{x})$, i.e. $\nabla I(\mathbf{x}', t) = \alpha(\mathbf{x}', t)\mathbf{u}$ for a scalar function α and a 2D vector $\mathbf{u}$, M is not invertible.

   Explain how this observation is related to the aperture problem.

   (c) Write down explicit expressions for the two components $\hat{v}_1$ and $\hat{v}_2$ of the minimizer in terms of m_{ij} and q_i.

   Note: $G * f$ denotes the convolution of a function $f : \mathbb{R}^2 \to \mathbb{R}$ with a kernel $G : \mathbb{R}^2 \to \mathbb{R}$ and is defined as

$$G * f = \int_{\mathbb{R}^2} G(\mathbf{x} - \mathbf{x}') f(\mathbf{x}') \mathrm{d}\mathbf{x}' \,.$$

2. **The Reconstruction Problem**

   The bundle adjustment (re-)projection error for N points $\mathbf{X}_1$,...,$\mathbf{X}_N$ is

$$E(R, \mathbf{T}, \mathbf{X}_1, ..., \mathbf{X}_N) = \sum_{j=1}^{N} \left(\|\mathbf{x}_1^j - \pi(\mathbf{X}_j)\|^2 + \|\mathbf{x}_2^j - \pi(R\mathbf{X}_j + \mathbf{T})\|^2 \right)$$

   (a) What dimension does the space of unknown variables have if ...
   - ... R is restricted to a rotation about the camera's y-axis?
   - ... the camera is only rotated, not translated?
   - ... the points $\mathbf{X}_j$ are known to all lie on one plane?