Multiple View Geometry: Exercise Sheet 7

Prof. Dr. Daniel Cremers,
Shenhan Qian, Simon Weber, Anna Ribic, and Tarun Yenamandra
Computer Vision Group, TU Munich
`Wednesdays 16:15–18:15 at Hörsaal 2, "Interims I" (5620.01.102), and on RBG Live`

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1. **Projection and Essential Matrix**

   Suppose two projection matrices $\Pi = [R, T]$ and $\Pi' = [R', T'] \in \mathbb{R}^{3\times 4}$ are related by a common transformation H of the form

   $$H = \begin{bmatrix} I & 0 \\ v^\top & v_4 \end{bmatrix} \in \mathbb{R}^{4\times 4} \quad \text{where } v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

   That is, $[R, T]H \sim [R', T']$ are equal up to scale.

   Show that Π and Π' give the same essential matrices ($E = \hat{T}R$ and $E' = \hat{T}'R'$) up to a scale factor.

2. **Coimages of Points and Lines**

   Suppose $p_1, p_2 \in \mathbb{R}^3$ are two points on the line $L \subset \mathbb{R}^3$. Let $x_1, x_2 \in \mathbb{R}^3$ be the images of the points p_1, p_2 in homogeneous coordinates, respectively, and let $l \in \mathbb{R}^3$ be a vector that spans the coimage of the line L. All vectors are given in the image coordinate system.
   Furthermore suppose $L_1, L_2 \subset \mathbb{R}^3$ are two lines intersecting in the point $p \in \mathbb{R}^3$. Let $x \in \mathbb{R}^3$ be the image of the point p in homogeneous coordinates and let $l_1, l_2 \in \mathbb{R}^3$ be vectors that span the coimages of the lines L_1, L_2, respectively.

   Draw a picture and convince yourself of the following relationships:

   (a) Show that
   $$l \sim \hat{x_1} x_2, \qquad x \sim \hat{l_1} l_2,$$

   (b) Show that there exist $r, s, u, v \in \mathbb{R}^3$ such that,
   $$l_1 \sim \hat{x} u, \qquad l_2 \sim \hat{x} v, \qquad x_1 \sim \hat{l} r, \qquad x_2 \sim \hat{l} s$$

   where $\sim$ means equivalence in the sense of homogeneous coordinates.

3. **Rank Constraints**

   Let $x_1, x_2 \in \mathbb{R}^3$ be two image points in homogeneous coordinates with projection matrices $\Pi_1, \Pi_2 \in \mathbb{R}^{3\times 4}$. Show that the rank constraint

   $$\text{rank} \begin{pmatrix} \hat{x_1} \Pi_1 \\ \hat{x_2} \Pi_2 \end{pmatrix} \leqq 3$$

   ensures that x_1 and x_2 are images (projections) of the same three-dimensional point X.