



Multiple View Geometry: Exercise Sheet 8

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Wednesdays 16:15–18:15 at Hörsaal 2, "Interims I"
(5620.01.102), and on RBG Live

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Download the ICRA 2013 paper *Robust Odometry Estimation for RGB-D Cameras* by Kerl, Sturm and Cremers from the *Publications* sections on our webpage.¹ Read the paper and focus in particular on *III. Direct Motion Estimation*.

1. Image Warping

- (a) Look at the warping function $\tau(\xi, \mathbf{x})$ in Eq. (9). What do $\tau(\xi, \mathbf{x})$ and $r_i(\xi)$ look like at $\xi = \mathbf{0}$?
- (b) Prove that the derivative of $r_i(\xi)$ w.r.t. ξ at $\xi = \mathbf{0}$ is

$$\left. \frac{\partial r_i(\xi)}{\partial \xi} \right|_{\xi=\mathbf{0}} = \frac{1}{z} \begin{pmatrix} I_x f_x & I_y f_y \end{pmatrix} \begin{pmatrix} 1 & 0 & -\frac{x}{z} & -\frac{xy}{z} & z + \frac{x^2}{z} & -y \\ 0 & 1 & -\frac{y}{z} & -z - \frac{y^2}{z} & \frac{xy}{z} & x \end{pmatrix} \bigg|_{(x,y,z)^\top = \pi^{-1}(\mathbf{x}_i, Z_1(\mathbf{x}_i))}$$

To this end, apply the chain rule multiple times and use the following identity:

$$\left. \frac{\partial T(g(\xi), \mathbf{p})}{\partial \xi} \right|_{\xi=\mathbf{0}} = (\text{Id}_3 \quad -\hat{\mathbf{p}}) \in \mathbb{R}^{3 \times 6}.$$

Note: The notation $\partial f(x)/\partial x$ denotes the Jacobian matrix including all first-order partial derivatives, where the number of rows is the number of dimensions of $f(x)$, and the number of columns is the number of dimensions of x .

- (c) Following the derivation in (b), determine the derivative for arbitrary ξ

$$\left. \frac{\partial r_i(\Delta \xi \circ \xi)}{\partial \Delta \xi} \right|_{\Delta \xi=\mathbf{0}}$$

where $\circ$ is defined by

$$\xi_1 \circ \xi_2 := \log \left(\exp(\hat{\xi}_1) \cdot \exp(\hat{\xi}_2) \right)^\vee.$$

$\vee: \mathfrak{se}(3) \rightarrow \mathbb{R}^6$ is the inverse of the hat transform.

Hint: Rewrite the problem such that you can make use of part b).

¹https://cvg.cit.tum.de/_media/spezial/bib/kerl13icra.pdf

2. Image Pyramids

In order to handle large translational and rotational motions, a coarse-to-fine scheme is applied in the paper. To go from one level l to $l + 1$, the images $I^{(l)}$ (intensity) and $D^{(l)}$ (depth) are downsampled by averaging over intensities or valid depth values, respectively:

$$I^{(l+1)}(n, m) := \frac{1}{4} \cdot \sum_{(n', m') \in O(n, m)} I^{(l)}(n', m')$$

$$O(n, m) = \{(2n, 2m), (2n + 1, 2m), (2n, 2m + 1), (2n + 1, 2m + 1)\}$$

$$D^{(l+1)}(n, m) := \frac{1}{|O_d(n, m)|} \cdot \sum_{(n', m') \in O_d(n, m)} D^{(l)}(n', m')$$

$$O_d(n, m) = \{(n', m') \in O(n, m) : D(n', m') \neq 0\}$$

How does the camera matrix K change from level l to $l + 1$? Write down $f_x^{(l+1)}$, $f_y^{(l+1)}$, $c_x^{(l+1)}$ and $c_y^{(l+1)}$ in terms of $f_x^{(l)}$, $f_y^{(l)}$, $c_x^{(l)}$ and $c_y^{(l)}$.

The following problem requires parts of the content from Lecture on Wednesday, 26.07. You can start them after that lecture.

3. Optimization for Normally Distributed $p(r_i)$

- (a) Confirm that a normally distributed $p(r_i)$ with a uniform prior on the camera motion leads to normal least squares minimization. To this end, use

$$p(r_i|\xi) = p(r_i) = A \exp\left(-\frac{r_i^2}{\sigma^2}\right)$$

to show that with a constant prior $p(\xi)$, the maximum a posteriori estimate is given by

$$\xi_{\text{MAP}} = \arg \min_{\xi} \sum_i r_i(\xi)^2.$$

- (b) Explicitly show that the weights

$$w(r_i) = \frac{1}{r_i} \frac{\partial \log p(r_i)}{\partial r_i}$$

are constant for normally distributed $p(r_i)$.

- (c) Show that in the case of normally distributed $p(r_i)$ the update step $\Delta\xi$ can be computed as

$$\Delta\xi = -\left(J^\top J\right)^{-1} J^\top \mathbf{r}(\mathbf{0}).$$