Combinatorial Optimization in Computer Vision

Chapter 8: Labelling with Ordering Constraint

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Labelling Problems with Ordering Constraint

Problem: In general, the labelling problem with ordering constraint deals with the assignment of a label in $\mathcal{L}$ for each pixel of an image such that the final labelling $x : V \rightarrow \mathcal{L}$ satisfies certain geometric constraints.

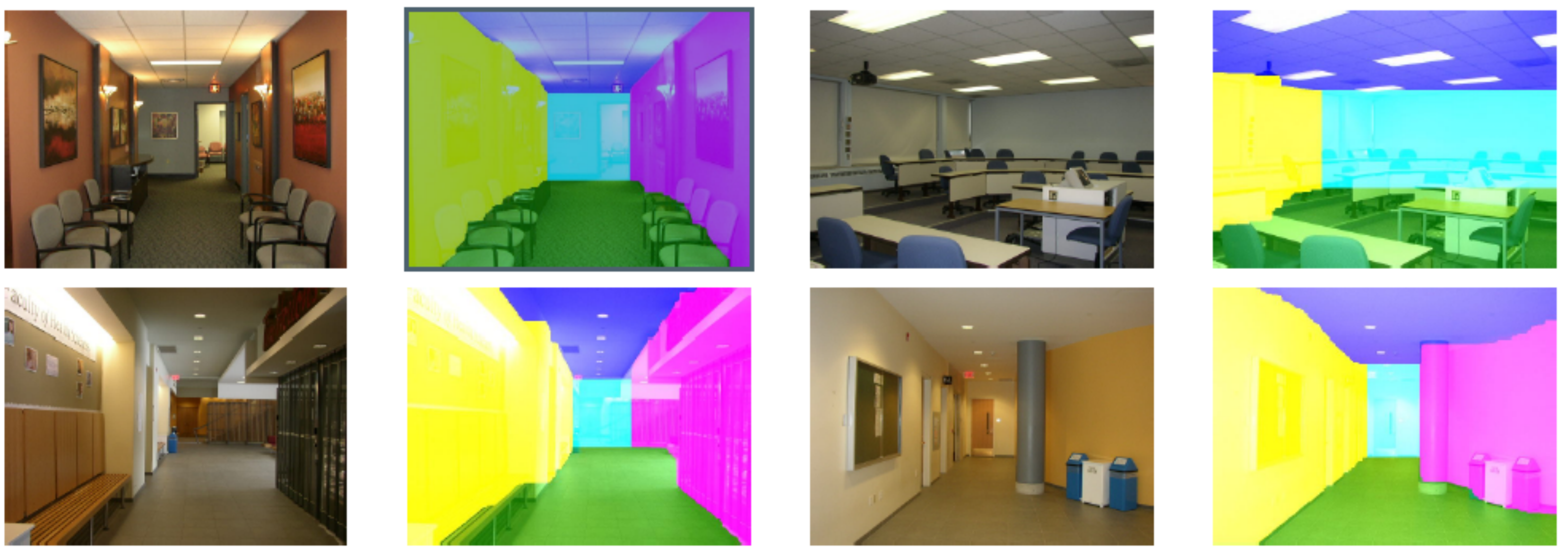
Examples include:

- $\mathcal{L}$ comprises parts of a class of objects. Then one might want to impose constraints as: „a *car wheel* cannot be below a *car roof*".
- „The *sky* is above the *street*."

Geometric Scene Labelling

Here, we will discuss an approach by Liu, Samarabandu and Veksler who consider the specific problem of finding a multilabel segmentation $x : V \to \mathcal{L}$ with

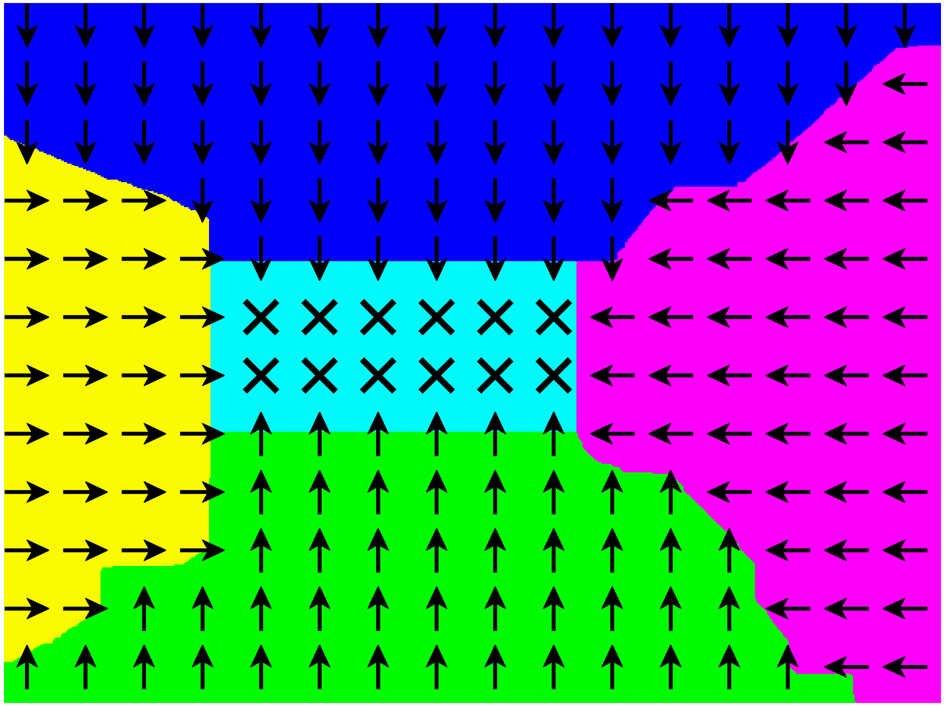
$$\mathcal{L} := \{C = \text{center}, L = \text{left}, R = \text{right}, B = \text{bottom}, T = \text{top}\}.$$



Constraints on Geometric Scene Labels

There are some obvious constraints on these labels like

- „left“ cannot appear right from „center“ or „right“
- „top“ cannot appear below „center“ or „bottom“



MRF Model

The resulting problem is formulated using an MRF model $X = (X_v)_{v \in V}$ where V is the set of pixels and the X_v take values in $\mathcal{L}$.

The corresponding Gibbs potential is given by

$$E(x) = \underbrace{\sum_{v \in V} \phi_v(x(v))}_{\text{data term}} + \underbrace{\sum_{v \sim w} a_{vw} \psi(x(v), x(w))}_{\text{regularizer}} + \underbrace{\sum_{v \sim w} \theta_{v,w}(x(v), x(w))}_{\text{constraints}} .$$

Here, the data term is learned from a database of images and the Potts model is used for the regularizer.

Constraints in the Gibbs Potential

The constraints are incorporated into the Gibbs potential by penalizing two neighboring label assignments, which violate a constraint, with penalty ∞.

For example, if v is the left neighbor pixel of w, then the table for $\theta(\alpha, \beta)$ looks like (the label of v is indicated in the row)

	L	R	C	T	B
L	0	∞	0	0	0
R	∞	0	∞	∞	∞
C	∞	0	0	∞	∞
T	∞	0	∞	0	∞
B	∞	0	∞	∞	0

Minimization of Gibbs Potential

The Gibbs potential

$$E(x) = \underbrace{\sum_{v \in V} \phi_v(x(v))}_{\text{data term}} + \underbrace{\sum_{v \sim w} a_{vw} \psi(x(v), x(w))}_{\text{regularizer}} + \underbrace{\sum_{v \sim w} \theta_{v,w}(x(v), x(w))}_{\text{constraints}}.$$

can be approximately minimized using the expansion algorithm.

However, in this case, there is no quality guarantee (because the pairwise potential is not a metric).

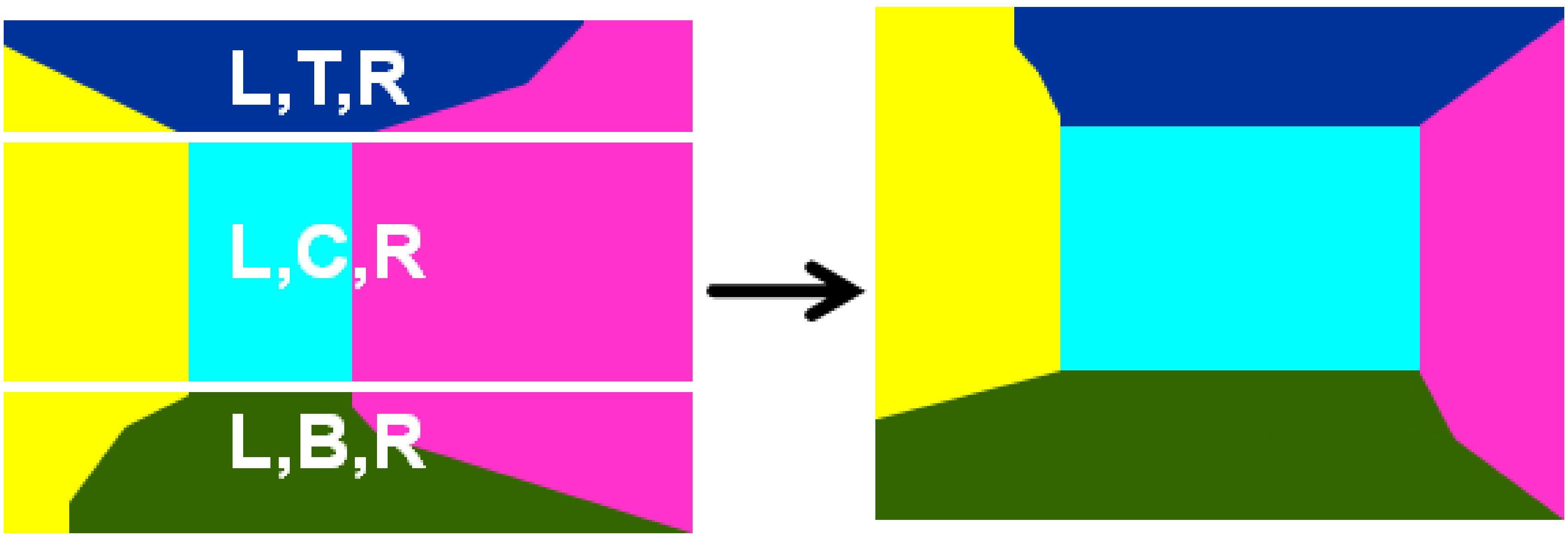
Order-Preserving Moves

It turns out that the expansion algorithm converges to bad local minima in this example. Thus, a new class of moves is introduced, the so-called order-preserving moves.

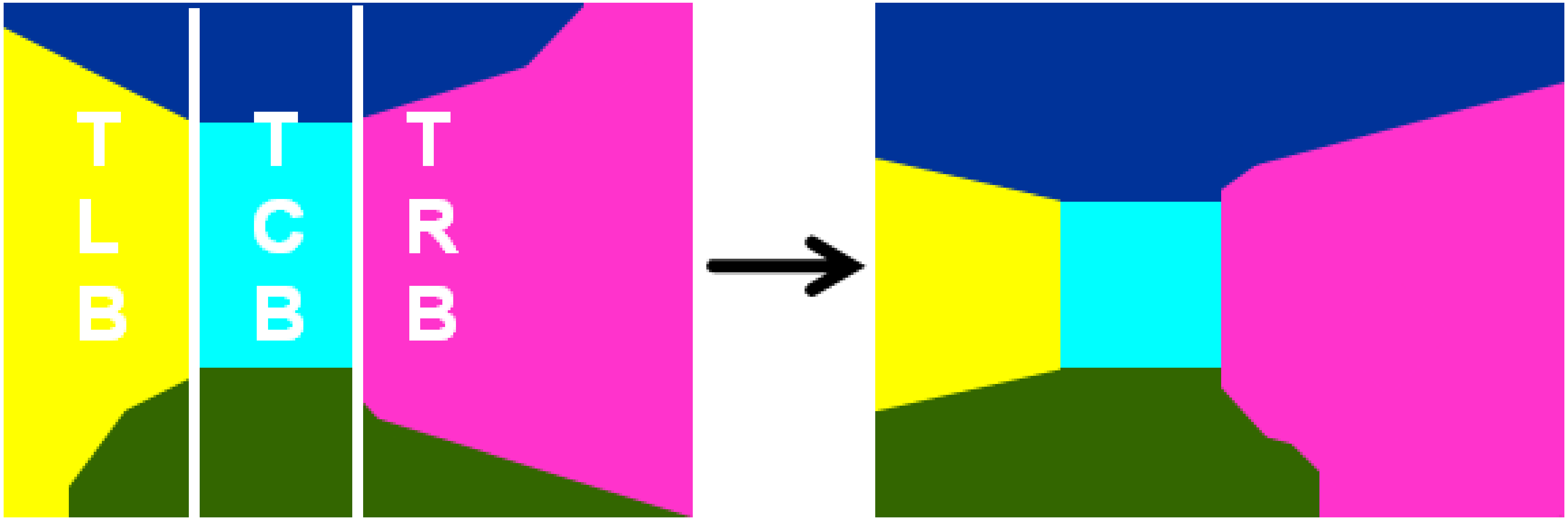
Horizontal Moves: Region C can change in horizontal but not in vertical direction.

Vertical Moves: Region C can change in vertical but not in horizontal direction.

Horizontal Moves



Vertical Moves



Order-Preserving Move Algorithm

1. Initialize $x(v) \leftarrow C \quad \forall v \in V.$

2. $x_h \leftarrow \text{optimal horizontal move from } x$

   $x_v \leftarrow \text{optimal vertical move from } x$

3. If $E(x_h) < E(x_v)$, set $x \leftarrow x_h$, else $x \leftarrow x_v$.

4. Repeat

   $x_h \leftarrow \text{optimal horizontal move from } x$

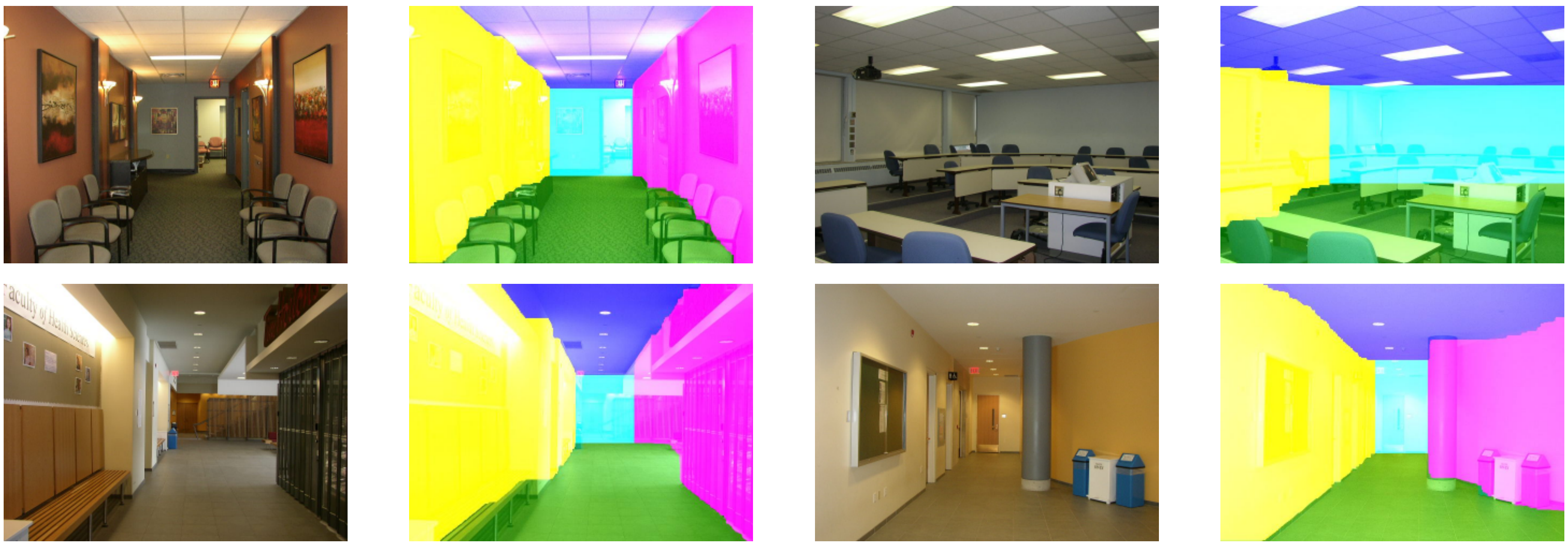
   $x \leftarrow \text{optimal vertical move from } x_h$

   until $E(x)$ stops decreasing.

Comments

- Similarly to the expansion algorithm, this method produces a local optimum.
- Unfortunately, no quality guarantee is available.
- The core step is the search for the optimal horizontal or vertical move. In each of these, a multilabel problem over more than 2 labels has to be solved. A result by Schlesinger and Flach shows that this can be done in polynomial time by computing a cut in an appropriate graph.

Results: Indoor Images



Results: Outdoor Images

Scene Walk using the Geometric Scene Labels

Summary

- Frequently we want to impose geometric constraints on labellings.

- Liu, Samarabandu and Veksler propose a move-make algorithm for computing local optima of the corresponding MRF energy.

- Similarly to the expansion algorithm, the core step consists in computing the optimal move from a given labelling. Again, this is solved by computing a cut in an appropriate graph.