Variational Methods for Computer Vision: Exercise Sheet 8

Exercise: 16 December 2011

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $Q := [0,1] \times [0,1]$ be a rectangular area and let $V = (v(x,y), u(x,y))$ be a differentiable vector field defined on Q.

   (a) Prove Green's theorem for Q using the fundamental theorem of calculus hence show that:

   $$\int_Q v_x(x,y) - u_y(x,y)\; dxdy = \oint_{\partial Q} V(s) d\vec{s}$$

   Assume the boundary curve ∂Q to be be oriented counter clockwise.

   (b) Let $\Omega \subset \mathbb{R}^2$ be an area that can be represented as a disjoint union of a finite number of squares $Q_1, ..., Q_n$. Prove that the Green theorem also holds for Ω.

2. In the lecture the piecewise constant Mumford-Shah functional is written as follows:

   $$E(u_i, C) = \sum_{i=1}^{n} \int_{\Omega_i} (I(x) - u_i)^2\; dx + \nu |C|$$

   Prove that by merging two regions Ω_1 and Ω_2 the energy E changes by :

   $$\delta E = \frac{A_1 A_2}{A_1 + A_2} (u_1 - u_2)^2 - \nu \delta C$$

   Where A_i denotes the area of the regions in pixels, u_i the respective mean values and δC the length of the interface of both regions.

3. Let $\Omega = [-5; 5] \times [-5; 5]$ be a rectangular area and let $I : \Omega \to [0; 1]$ be an image given by :

   $$I(x,y) = \begin{cases} 1 & \text{if} \quad x^2 + y^2 \leq 1 \\ 0 & \text{else} \end{cases}$$

   Furthermore let $C : [0,1] \to \Omega$ be a curve represented by a circle centered at the origin having radius r.

   (a) Write down the Gateaux-Derivatives of the Mumford Shah functional for 2 regions for the two cases $r > 1$ and $r \leq 1$.

   (b) Show that the Gateau derivative at $r = 0$ is not continuous. Why is $\nu \leq 1$ a good choice in order to obtain good segmentation results. What is the ideal choice for ν in our example?

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. No practical exercise this week. Finish implementing the practical exercise of sheet 6.