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Fakultät für Informatik
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Statistical Methods and Learning in Computer Vision
Winter term 2012

Homework Assignment 1 — October 30th, 2012

Exercise 1. (σ-algebras)

Let Ω be a given set and $B \subset \Omega$. Show that the set

$$\mathcal{A} := \{A \subset \Omega | B \subset \Omega \text{ or } B \cap A = \emptyset\} \tag{1}$$

is a σ-algebra.

Exercise 2. (Image Measures)

We want to measure the probability of throwing twice the same number with two regular dice, e.g. $(1,1)$ or $(6,6)$. Define the state space Ω, the σ-algebra over Ω and the uniform probability measure P. Then define a random variable X, which indicates if both dice show the same result. Then use the image measure P_X to obtain the probability of throwing the same result with both dice.

Exercise 3. (Conditional and Marginal Probabilities)

In 1975, in Germany 0.5% of the population were sick with Tbc (Tuberculose). A special test for Tbc is known to yield the correct diagnosis for 90% of the sick people and 99% of the healthy people.

a) Proof Bayes formula for two sets A and B:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{2}$$

b) Mr. M. has taken part in the Tbc-test. The test result says that he has Tbc. Compute the conditional probability that Mr M. is really sick with Tbc.

Exercise 4. (Uncorrelated and Independent Random Variables)

Let $X : (\Omega, \mathcal{B}, P) \to (\Omega, \mathcal{B}, P)$, $X(\omega) = \omega$ be a random variable. Let $Y := X^2$ be a second random variable.

a) Show that X and Y are uncorrelated by computing their covariance.

b) Show that X and Y are not independent.

So we have shown that uncorrelated random variables are not necessarily independent.

Exercise 5. (Variance of the Sum of Random Variables)

Let X and Y by two random variables. Proof for the variance of the sum of these that

$$V(X+Y) = V(X) + V(Y) + 2Kov(X,Y) \tag{3}$$

What happens if X and Y are uncorrelated?

The next exercise class will take place on **November 6th, 2012**.

For downloads of slides and of homework assignments and for further information on the course see

`http://vision.in.tum.de`
