TU MÜNCHEN FAKULTÄT FÜR INFORMATIK DR. CLAUDIA NIEUWENHUIS ENO TÖPPE

Statistical Methods and Learning in Computer Vision Winter term 2012

Homework Assignment 1 — October 30th, 2012

Exercise 1. (σ -algebras)

Let Ω be a given set and $B \subset \Omega$. Show that the set

$$\mathcal{A} := \{ A \subset \Omega | B \subset \Omega \text{ or } B \cap A = \emptyset \} \tag{1}$$

is a σ -algebra.

Exercise 2. (Image Measures)

We want to measure the probability of throwing twice the same number with two regular dice, e.g. (1,1) or (6,6). Define the state space Ω , the σ -algebra over Ω and the uniform probability measure P. Then define a random variable X, which indicates if both dice show the same result. Then use the image measure P_X to obtain the probability of throwing the same result with both dice.

Exercise 3. (Conditional and Marginal Probabilities)

In 1975, in Germany 0.5% of the population were sick with Tbc (Tuberculose). A special test for Tbc is known to yield the correct diagnosis for 90% of the sick people and 99% of the healthy people.

a) Proof Bayes formula for two sets A and B:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{2}$$

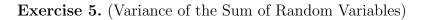
b) Mr. M. has taken part in the Tbc-test. The test result says that he has Tbc. Compute the conditional probability that Mr M. is really sick with Tbc.

Exercise 4. (Uncorrelated and Independent Random Variables)

Let $X:(\Omega,\mathcal{B},P)\to (\Omega,\mathcal{B},P),$ $X(\omega)=\omega$ be a random variable. Let $Y:=X^2$ be a second random variable.

- a) Show that X and Y are uncorrelated by computing their covariance.
- b) Show that X and Y are not independent.

So we have shown that uncorrelated random variables are not necessarily independent.



Let X and Y by two random variables. Proof for the variance of the sum of these that

$$V(X+Y) = V(X) + V(Y) + 2Kov(X,Y)$$

$$\tag{3}$$

What happens if X and Y are uncorrelated?

The next exercise class will take place on November 6th, 2012.

For downloads of slides and of homework assignments and for further information on the course see