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Statistical Methods and Learning in Computer Vision
Winter term 2012

Homework Assignment 3 — November 21th, 2012

Exercise 1. (Kalman Filter)

For bivariate normal distributions the following **theorem** holds:

$$p(x_1, x_2) = \mathcal{N}\left((\mu_1, \mu_2), \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right)$$

if and only if

$$p(x_1|x_2) = \mathcal{N}(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2),\ \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}) \quad \text{and} \quad p(x_2) = \mathcal{N}(\mu_2, \Sigma_{22})$$

Now, consider a Kalman filter. Using the notation in the lecture, the probability $p(x_k|z_1, ..., z_k)$ is a Gaussian of the form $\mathcal{N}(\tilde{\mu_k}, \tilde{\Sigma_k})$. Derive $\tilde{\mu_k}$ and $\tilde{\Sigma_k}$. To this end, apply the above theorem to the conditional probability $p(z_k|x_k, z_1, ..., z_{k-1}) = \mathcal{N}(Hx_k, R)$ and the marginal probability $p(x_k|z_1, ..., z_{k-1}) = \mathcal{N}(F\mu_k, F\Sigma_k F^T + Q) = \mathcal{N}(\bar{\mu_k}, \bar{\Sigma_k})$ in order to get the joint probability $p(z_k, x_k)$. Then, apply the theorem again to $p(z_k, x_k)$ in order to derive $p(x_k|z_1, ..., z_k)$.

The next exercise class will take place on **November 27th, 2012**.

For downloads of slides and of homework assignments and for further information on the course see

`http://vision.in.tum.de`