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# Statistical Methods and Learning in Computer Vision
# Winter term 2012

## Homework Assignment 6 — January 15th, 2013

**Exercise 1.** (Linear Discriminant Analysis)

To be more precise, Linear Discriminant Analysis is a special case of Fisher's discriminant $w = \arg\max_w \left(\frac{w^T S_B w}{w^T S_W w}\right)$. In its original form Linear Discriminant Analysis assumes that the samples of the classes are distributed normally with each class having the same covariance matrix $\Sigma$. Linear Discriminant Analysis then strives to find a separating hyperplane, i.e. a linear discriminant.

a) So, assume a Gaussian mixture model (i.e. the probability of a given sample $x$ belonging to class $k$ is given by $P(C = k|X = x) = \frac{\mathcal{N}(\Sigma_k, \mu_k)\pi_k}{\sum_i \mathcal{N}(\Sigma_i, \mu_i)\pi_i}$) for two classes $k = \{1, 2\}$ and $\Sigma_1 = \Sigma_2$. Now, show that all the points $x$ where the posterior probabilities for both classes are equal, i.e.

$$\log \frac{P(C = 1|X = x)}{P(C = 2|X = x)} = 0$$

lie on a hyperplane (they satisfy $w^T x = d$). This is the LDA solution in its original form.

b) Show that for two classes Fisher's discriminant yields

$$w = c \cdot (\Sigma_1^{-1} + \Sigma_2^{-1})(\mu_2 - \mu_1).$$

for a constant $c$.

Conclude from this result that in the case of two Gaussian distributions $\mathcal{N}(\Sigma_i, \mu_i)$, $i \in \{1, 2\}$ with $\Sigma_1 = \Sigma_2$ Fisher's discriminant yields the normal direction of the class separating hyperplane in a).