TU München
Fakultät für Informatik
Dr. Claudia Nieuwenhuis
Eno Töppe

Statistical Methods and Learning in Computer Vision
Winter term 2012

Homework Assignment 7 — January 15th, 2013

Exercise 1. (Independent Component Analysis)

The goal of this exercise is to motivate and study the preprocessing steps of ICA. Given are two sets of equally distributed random variables $s_1(t)$ and $s_2(t)$ (e.g. two signals over time) with respective probability distribution functions $p_{s_1}(.)$ and $p_{s_2}(.)$. In the following we will omit the dependence on t.

a) For ICA we assume unit variance and mean value 0 for the independent components s_i. Which equally distributed pdf on $\mathbb{R}$ has these properties? How does the marginal probability $p(s_1, s_2)$ look like?

b) The independent signals s_i are transformed to x_i by the following mixing matrix

$$A = \begin{pmatrix} 5 & 10 \\ 10 & 2 \end{pmatrix}.$$

Sketch the transformation of the sample space of s_i graphically. What effect does A have on the independence and correlation of the variables x_i?

c) Let Σ be the covariance matrix of the random variables $\mathbf{x}(t)$. Furthermore, let matrix P and diagonal matrix D be defined such that $P\Sigma P^T = D$. Now *whitening* is performed on the *centered* values $\mathbf{x} = A\mathbf{s}$ (see lecture 7). To this end, we multiply $\mathbf{x}$ by $V := D^{-\frac{1}{2}}P^T$ from the left. Show that the variables $\mathbf{z} = V\mathbf{x} = VA\mathbf{s}$ are indeed whitened (i.e. $Cov(\mathbf{z}(t)) = Id$) and that whitening is only defined up to a rotation, i.e. a matrix U for which $UU^T = I$. Why is, therefore, whitening not sufficient to get hold of the independent components? Explain this informally by sketching similarly to part b) the effects of whitening and centering on the transformed sample space.