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Statistical Methods and Learning in Computer Vision
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Homework Assignment 9 — January 31th, 2013

Exercise 1. (Euler-Lagrange Equation)

Given a functional of the form

$$E(u) = \int \mathscr{L}(a, b, x)dx = \int \mathscr{L}(u, \nabla u, x)dx$$

a necessary condition for a point u^* being a local minimum of the functional E is given by the Euler-Lagrange equation:

$$\begin{aligned} \frac{\partial \mathscr{L}(u^*, \nabla u^*, x)}{\partial a} - \operatorname{div} \frac{\partial \mathscr{L}(u^*, \nabla u^*, x)}{\partial b} &= \\ \frac{\partial \mathscr{L}(u^*, \nabla u^*, x)}{\partial a} - \sum_i \frac{d}{dx_i} \frac{\partial \mathscr{L}(u^*, \nabla u^*, x)}{\partial b_i} &= 0 \quad \forall x \in U \end{aligned} \tag{1}$$

where $U \subset \mathbb{R}^n$.

You will learn more about the equation in the lecture on Thursday. Calculate the Euler-Lagrange equation for the *Inpainting* functional

$$E(u) = \int_\Omega (1 - 1_M(x))||u(x) - I(x)||^2 + ||\nabla u(x)||dx$$

Where 1_M is the characteristic function of the set M.

To make it easier, calculate the respective terms of equation (1) separately and put them together later on.