

Statistical Methods and Learning in Computer Vision
Winter term 2012

Homework Assignment 9 — January 31th, 2013

Exercise 1. (Euler-Lagrange Equation)

Given a functional of the form

$$E(u) = \int \mathcal{L}(a, b, x) dx = \int \mathcal{L}(u, \nabla u, x) dx$$

a necessary condition for a point u^* being a local minimum of the functional E is given by the Euler-Lagrange equation:

$$\begin{aligned} \frac{\partial \mathcal{L}(u, \nabla u, x)}{\partial a} - \operatorname{div} \frac{\partial \mathcal{L}(u, \nabla u, x)}{\partial b} &= \\ \frac{\partial \mathcal{L}(u, \nabla u, x)}{\partial a} - \sum_i \frac{d}{dx_i} \frac{\partial \mathcal{L}(u, \nabla u, x)}{\partial b_i} &= 0 \end{aligned} \quad (1)$$

You will learn more about the equation in the lecture on Thursday. Calculate the Euler-Lagrange equation for the *Inpainting* functional

$$E(u) = \int_{\Omega} (1 - 1_M(x)) \|u(x) - I(x)\|^2 + \|\nabla u(x)\| dx$$

Where 1_M is the characteristic function of the set M .

To make it easier, calculate the respective terms of equation (1) separately and put them together afterwards.