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Machine Learning for Robotics and Computer Vision
Winter term 2013

Homework Assignment 5
Topic: Kernel Methods
Tutorial December 13th, 2013

Exercise 1: Constructing kernels

Let k_1 and k_2 be kernels, $f : \mathbb{R}^n \to \mathbb{R}$ an arbitrary function . Show that we can construct new kernels via

1. $k(x_1, x_2) = k_1(x_1, x_2) + k_2(x_1, x_2)$
2. $k(x_1, x_2) = k_1(x_1, x_2)k_2(x_1, x_2)$
3. $k(x_1, x_2) = f(x_1)k_1(x_1, x_2)f(x_2)$
4. $k(x_1, x_2) = \exp(k_1(x_1, x_2))$
5. $k(x_1, x_2) = x_1^T A x_2$, A symmetric, positive semi-definit

Exercise 2: Polynomial kernel

Let $x_i, x_j \in \mathbb{R}^2$

1. Show (by induction) that $k_d(x_i, x_j) = (x_i^T x_j)^d$ is a kernel for every $d \geq 1$.
2. Find $\phi_d(x)$ such that $k_d(x_i, x_j) = \phi_d(x_i)^T \phi_d(x_j)$.
3. Find $\tilde{\phi}_2(x)$ for $\tilde{k}_2(x_i, x_j) = (x_i^T x_j + d)^2$ $(d > 0)$.

Exercise 3: Programming

Download the files exercise5.m, feature_test.m and feature_plot.m from the website.

1. Implement the function feature_test.
2. Test your code with the script exercise5.m and the following choices for ϕ:
   (a) $\phi(x, y) = \begin{pmatrix} x & y & x^2 + y^2 \end{pmatrix}^T$
   (b) $\phi(x, y) = \begin{pmatrix} x^2 & \sqrt{2}xy & y^2 \end{pmatrix}^T$
   (c) $\phi(x, y) = \begin{pmatrix} \sin(x)\cos(y) & \sin(x)\sin(y) & \cos(x) \end{pmatrix}^T$
   (d) $\phi(x, y) = \begin{pmatrix} x^3 & \sqrt{3}x^2y & \sqrt{3}xy^2 & y^3 \end{pmatrix}^T$
   (e) $\phi(x, y) = \begin{pmatrix} x^2 & \sqrt{2}xy & y^2 & \sqrt{2d}x & \sqrt{2d}y & d \end{pmatrix}^T$

   within appropriate domains and with different choices for n and c.

Hint:
Watching the following video helps understanding the mapping to the feature space:
`http://youtu.be/3liCbRZPrZA`.
