

Machine Learning for Robotics and Computer Vision
Winter term 2013

Homework Assignment 6

Topic: Sampling Methods
Tutorial January 31st, 2013

Exercise 1: Rejection Sampling

In this exercise, we show more carefully that rejection sampling does indeed draw samples from the desired distribution $p(z)$. Suppose that the proposal distribution is $q(z)$ and show that the probability of a sample value z being accepted is given by $\frac{\tilde{p}(z)}{kq(z)}$ where \tilde{p} is any unnormalized distribution that is proportional to $p(z)$, and the constant k is set to the smallest value that ensures $kq(z) \geq \tilde{p}(z)$ for all values of z . Note that the probability of drawing a value z is given by the probability of drawing this value from $q(z)$ times the probability of accepting that value given that it has been drawn. Make use of this, along with the sum and product rule of probability, to write down the normalized form for the distribution over z and show that it equals $p(z)$.

Exercise 2: Gibbs sampling algorithm

Show that the Gibbs sampling algorithm satisfies detailed balance:

$$\pi_i A_{ij} = \pi_j A_{ji}$$