Machine Learning for Computer Vision

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Lecturers

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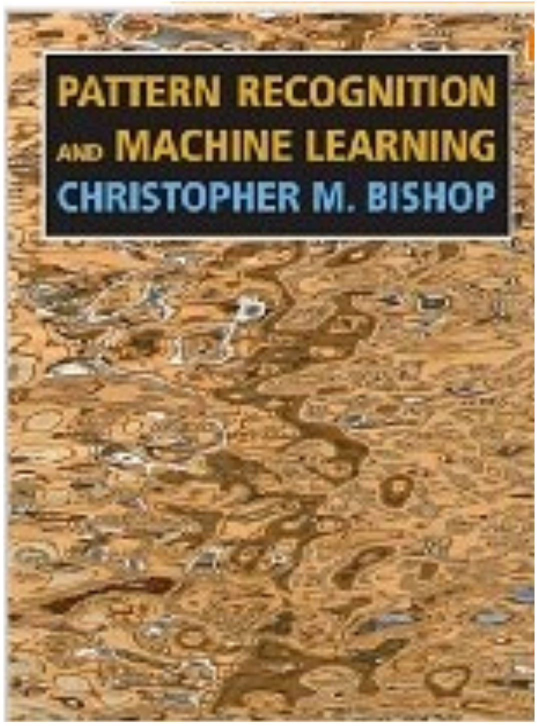
Class Schedule

Date	Topic
25.10.13	Introduction
8.11.13	Regression
15.11.13	Probabilistic Graphical Models I
22.11.13	Probabilistic Graphical Models II
29.11.13	Boosting
6.12.13	Kernel Methods
13.12.13	Gaussian Processes
20.12.13	Mixture Models and EM
10.1.14	Variational Inference
17.1.14	Sampling Methods
24.1.14	MCMC
31.1.14	Unsupervised Learning
7.2.14	Online Learning

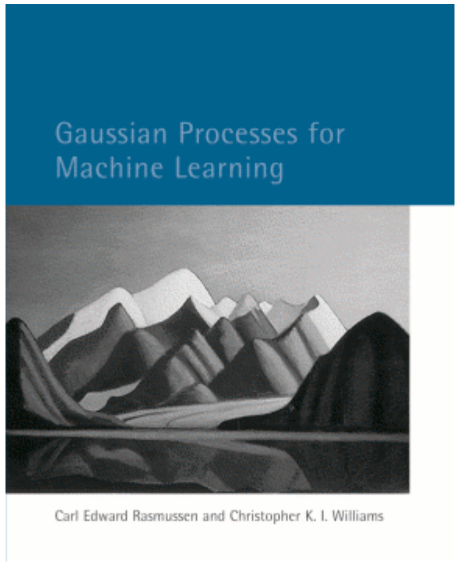
Literature

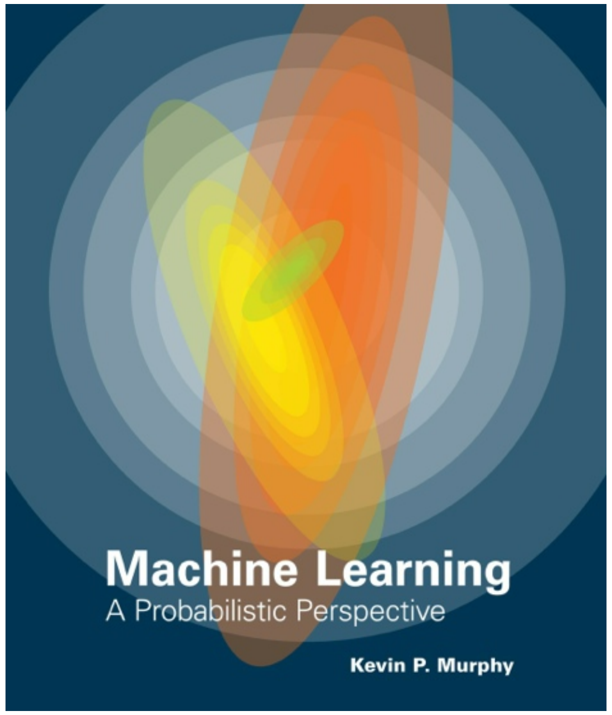


Recommended textbook for the lecture: Christopher M. Bishop: “Pattern Recognition and Machine Learning”

More detailed:

- “Gaussian Processes for Machine Learning” Rasmussen/Williams
- “Machine Learning - A Probabilistic Perspective” Murphy





The Tutorials

- Weekly tutorial classes
- Participation in tutorial classes and submission of solved assignment sheets is totally free
- The submitted solutions can be corrected and returned
- In class, you have the opportunity to present your solution
- Assignments will be theoretical and practical problems

The Exam

- No “qualification” necessary for the final exam
- Final exam will be oral
- From a given number of known questions, some will be drawn by chance
- Usually, from each part a fixed number of questions appears

Class Webpage

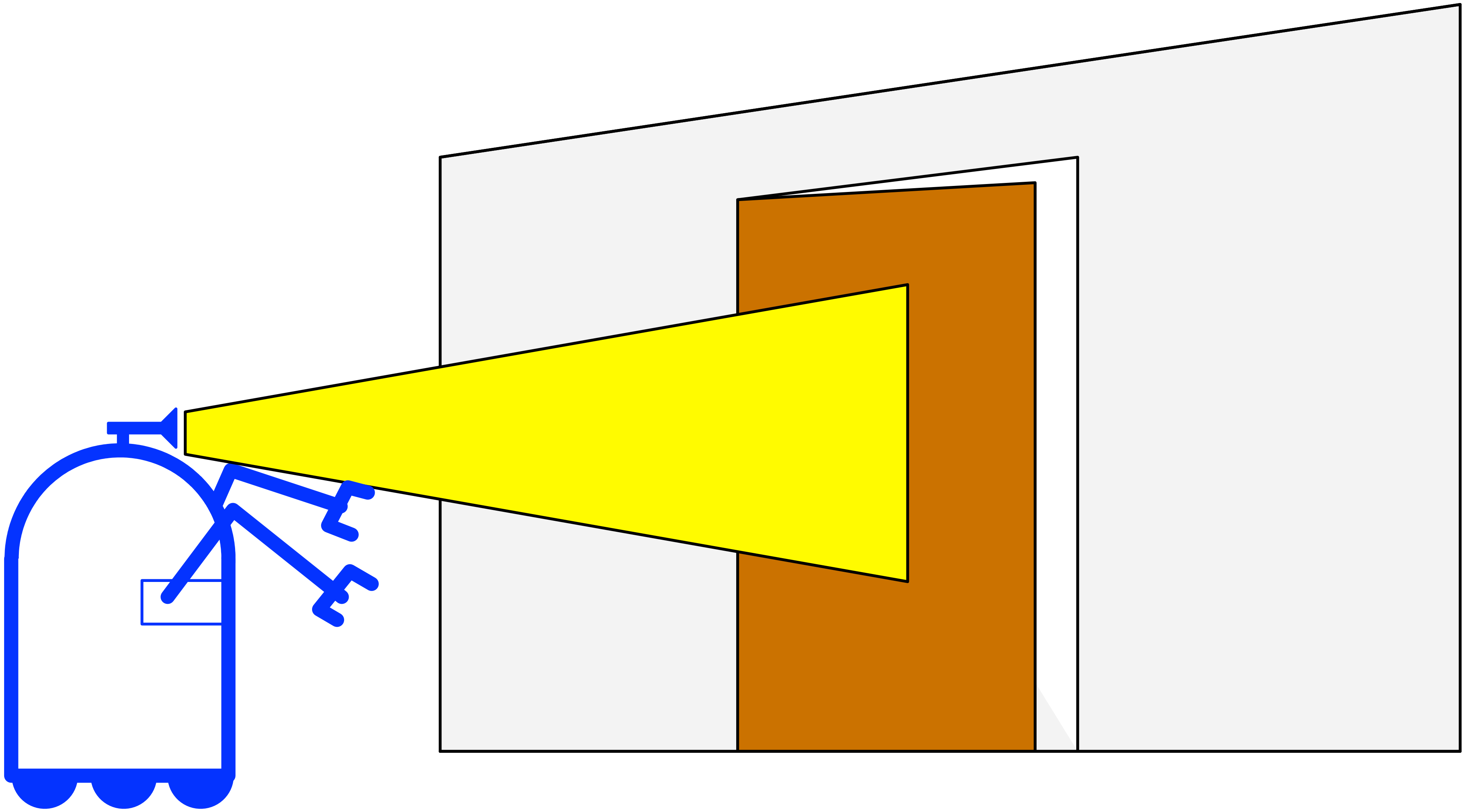
http://vision.in.tum.de/teaching/ws2013/ml_ws13

- Contains the slides and assignments for download
- Also used for communication, in addition to email list
- Some further material will be developed in class

1. Introduction to Learning and Probabilistic Reasoning

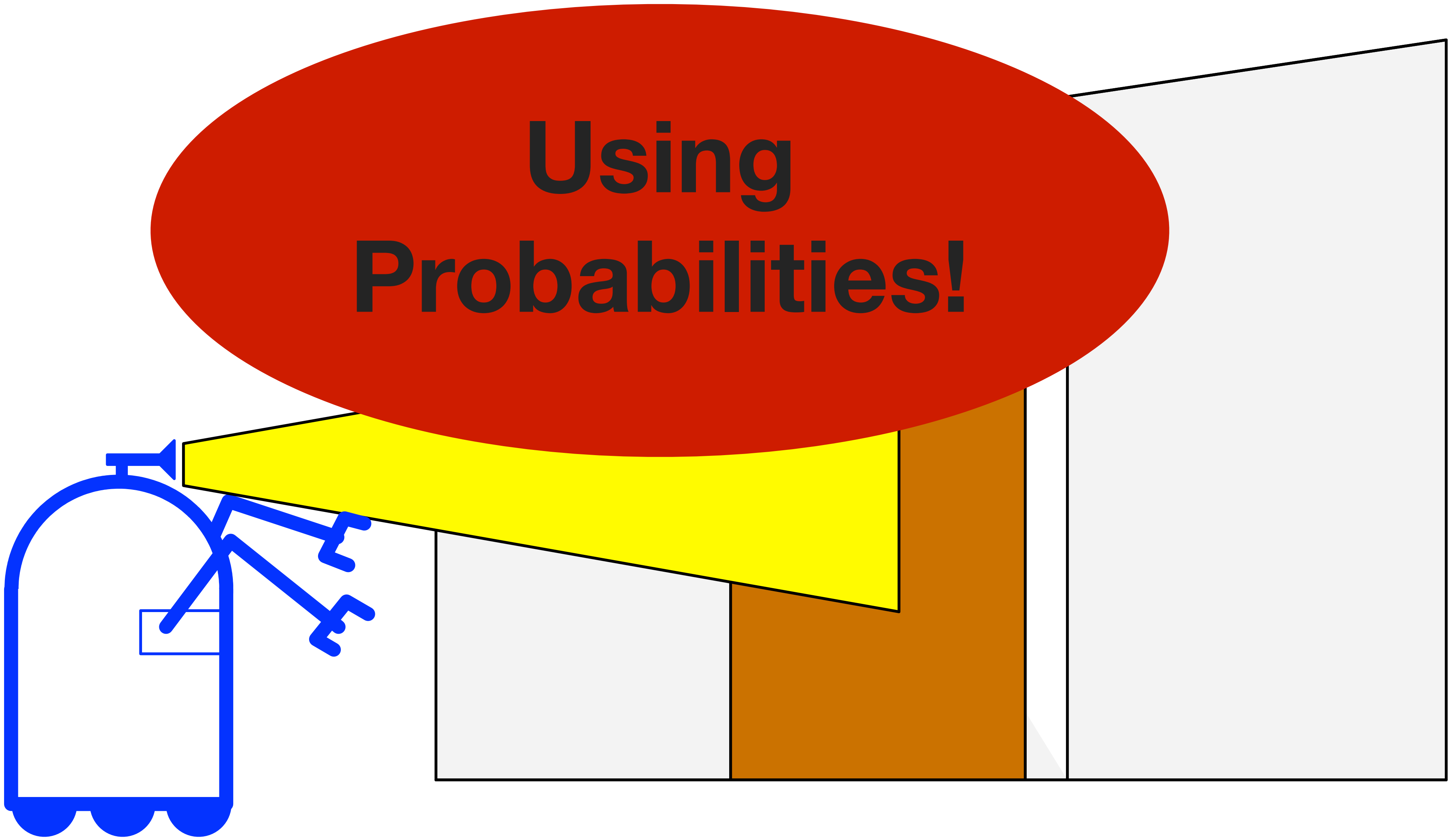
Motivation

Suppose a robot stops in front of a door. It has a sensor (e.g. a camera) to measure the state of the door (open or closed). **Problem**: the sensor may fail.



Motivation

Question: How can we obtain knowledge about the environment from sensors that may return incorrect results?



Basics of Probability Theory

Definition 1.1: A ***sample space*** $\mathcal{S}$ is a set of outcomes of a given experiment.

Examples:

a) Coin toss experiment: $\mathcal{S} = \{H, T\}$

b) Distance measurement: $\mathcal{S} = \mathbb{R}_0^+$

Definition 1.2: A ***random variable*** X is a function that assigns a real number to each element of $\mathcal{S}$.

Example: Coin toss experiment: $H = 1, T = 0$

Values of random variables are denoted with small letters, e.g.: $X = x$

Discrete and Continuous

If $\mathcal{S}$ is countable then X is a ***discrete*** random variable, else it is a ***continuous*** random variable.

The probability that X takes on a certain value x is a real number between 0 and 1. It holds:

$$\sum_x p(X = x) = 1$$

Discrete case

$$\int p(X = x) dx = 1$$

Continuous case

A Discrete Random Variable

Suppose a robot knows that it is in a room, but it does not know in ***which*** room. There are 4 possibilities:

Kitchen, Office, Bathroom, Living room

Then the random variable *Room* is discrete, because it can take on one of four values. The probabilities are, for example:

$$P(Room = \text{kitchen}) = 0.7$$

$$P(Room = \text{office}) = 0.2$$

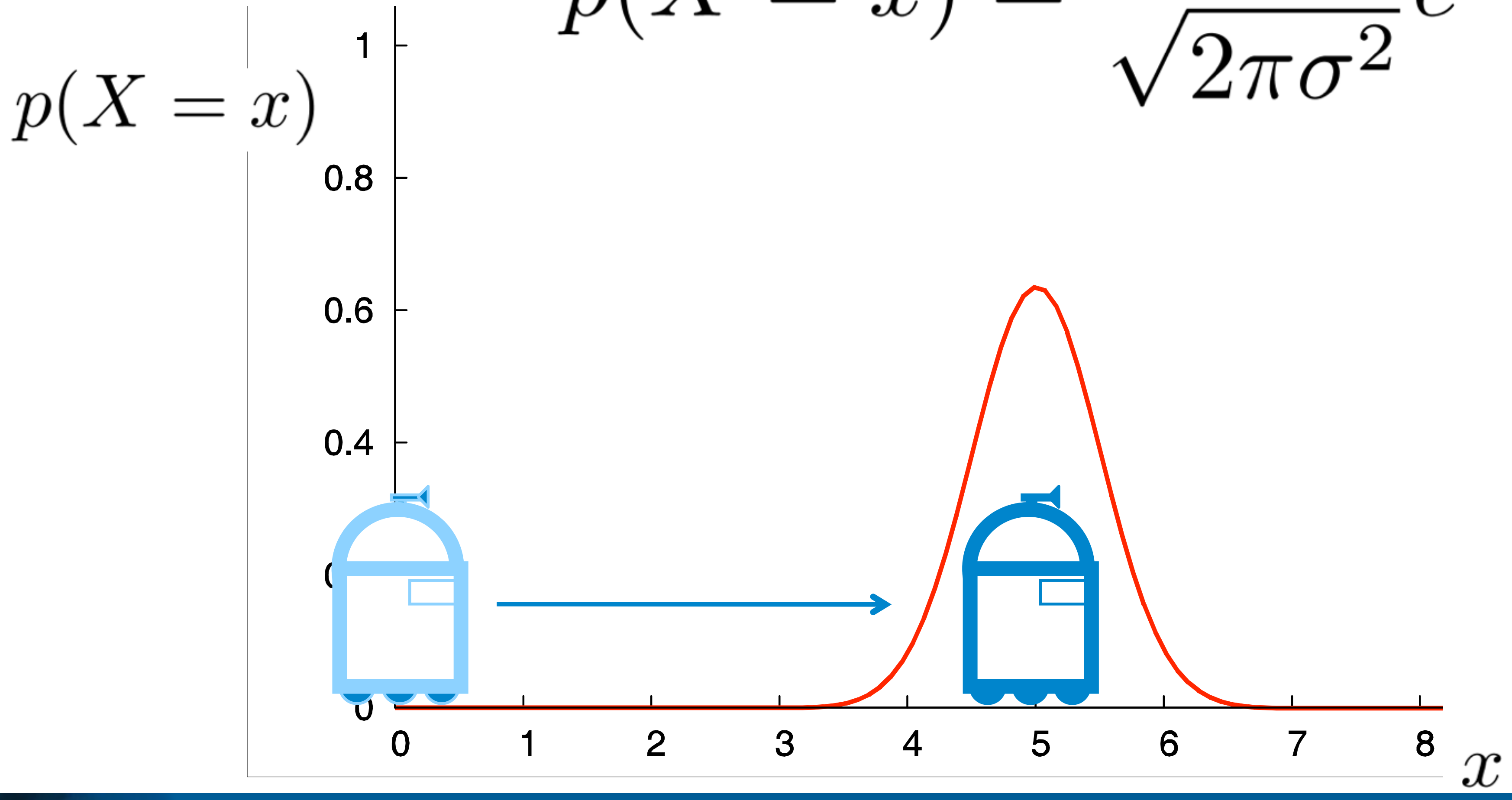
$$P(Room = \text{bathroom}) = 0.08$$

$$P(Room = \text{living room}) = 0.02$$

A Continuous Random Variable

Suppose a robot travels 5 meters forward from a given start point. Its position X is a continuous random variable with a ***Normal distribution***:

$$p(X = x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-5)^2}{\sigma^2}}$$



Shorthand:

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$$\mathcal{N}(x; \mu, \sigma^2)$$

Joint and Conditional Probability

The ***joint probability*** of two random variables X and Y is the probability that the events $X = x$ and $Y = y$ occur at the same time:

$$p(X = x \text{ and } Y = y)$$

Shorthand:

$$p(X = x) \rightarrow p(x)$$

$$p(X = x \text{ and } Y = y) \rightarrow p(x, y)$$

Definition 1.3: The ***conditional probability*** of X given Y is defined as:

$$p(X = x \mid Y = y) = p(x \mid y) := \frac{p(x, y)}{p(y)}$$

Independency, Sum and Product Rule

Definition 1.4: Two random variables X and Y are *independent* iff:

$$p(x, y) = p(x)p(y)$$

For independent random variables Xnd Ye have:

$$p(x \mid y) = \frac{p(x, y)}{p(y)} = \frac{p(x)p(y)}{p(y)} = p(x)$$

Furthermore, it holds:

$$p(x) = \sum_y p(x, y) \qquad p(x, y) = p(y \mid x)p(x)$$

"Sum Rule" **"Product Rule"**

Law of Total Probability

Theorem 1.1: For two random variables X and Y it holds:

$$p(x) = \sum_y p(x \mid y) p(y) \qquad p(x) = \int p(x \mid y) p(y) dy$$

Discrete case — Continuous case

The process of obtaining $p(x)$ from $p(x, y)$ by summing or integrating over all values of y is called

Marginalisation

Bayes Rule

Theorem 1.2: For two random variables X and Y it holds:

$$p(x \mid y) = \frac{p(y \mid x)p(x)}{p(y)}$$

"Bayes Rule"

Proof:

I. $$p(x \mid y) = \frac{p(x,y)}{p(y)}$$ *(definition)*

II. $$p(y \mid x) = \frac{p(x,y)}{p(x)}$$ *(definition)*

III. $$p(x,y) = p(y \mid x)p(x)$$ *(from II.)*

Bayes Rule: Background Knowledge

For $p(y \mid z) \neq 0$ it holds:

Background knowledge

$$p(x \mid y, z) = \frac{p(y \mid x, z)p(x \mid z)}{p(y \mid z)}$$

Shorthand: $p(y \mid z)^{-1} \rightarrow \eta$

“Normalizer”

$$p(x \mid y, z) = \eta \, p(y \mid x, z)p(x \mid z)$$

Computing the Normalizer

$$p(x \mid y) = \frac{p(y \mid x)p(x)}{p(y)}$$

Bayes rule

$$p(y) = \sum_x p(y \mid x)p(x)$$

Total probability

$$p(x \mid y) = \frac{p(y \mid x)p(x)}{\sum_{x'} p(y \mid x')p(x')}$$

$p(x \mid y)$ can be computed without knowing $p(y)$

Conditional Independence

Definition 1.5: Two random variables X and Y are *conditional independent* given a third random variable Z iff:

$$p(x, y \mid z) = p(x \mid z) p(y \mid z)$$

This is equivalent to:

$$p(x \mid z) = p(x \mid y, z)$$ and
$$p(y \mid z) = p(y \mid x, z)$$

Expectation and Covariance

Definition 1.6: The ***expectation*** of a random variable X is defined as:

$$E[X] = \sum_x x\ p(x)$$ (discrete case)

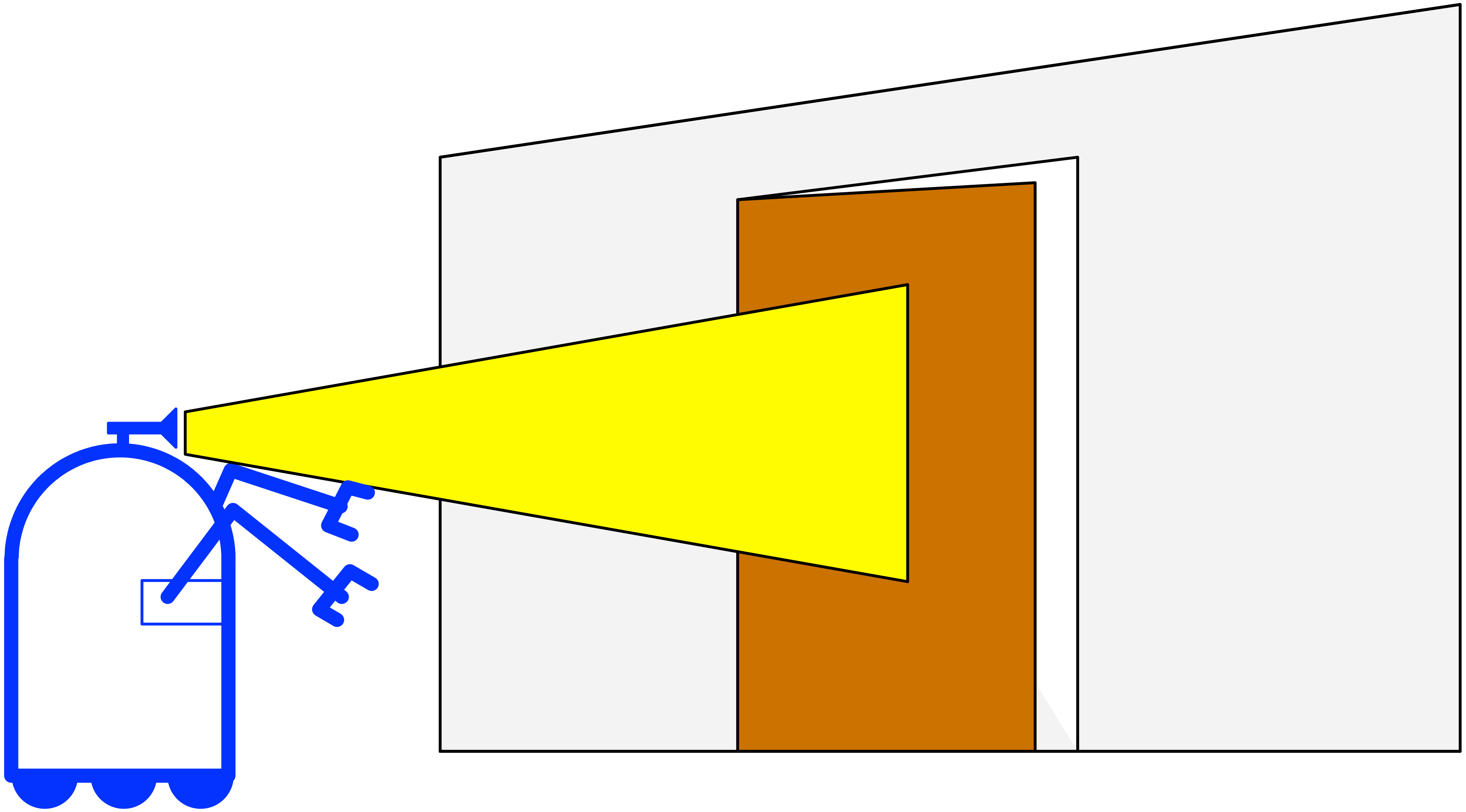
$$E[X] = \int x\ p(x) dx$$ (continuous case)

Definition 1.7: The ***covariance*** of a random variable X is defined as:

$$Cov[X] = E[(X - E[X])^2] = E[X^2] - E[X]^2$$

Mathematical Formulation of Our Example

We define two binary random variables:
z and open, where z is “light on” or “light off”. Our question is: What is $p(\text{open} \mid z)$?



Causal vs. Diagnostic Reasoning

- Searching for $p(\text{open} \mid z)$ is called ***diagnostic reasoning***
- Searching for $p(z \mid \text{open})$ is called ***causal reasoning***
- Often causal knowledge is easier to obtain
- Bayes rule allows us to use causal knowledge:

$$p(\text{open} \mid z) = \frac{p(z \mid \text{open})p(\text{open})}{p(z)}$$

$$= \frac{p(z \mid \text{open})p(\text{open})}{p(z \mid \text{open})p(\text{open}) + p(z \mid \neg\text{open})p(\neg\text{open})}$$

Example with Numbers

Assume we have this ***sensor model:***

$$p(z \mid \text{open}) = 0.6 \qquad p(z \mid \neg\text{open}) = 0.3$$

and: $p(\text{open}) = p(\neg\text{open}) = 0.5$ ***"Prior prob."***

then:

$$\begin{aligned} p(\text{open} \mid z) &= \frac{p(z \mid \text{open})p(\text{open})}{p(z \mid \text{open})p(\text{open}) + p(z \mid \neg\text{open})p(\neg\text{open})} \\ &= \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{2}{3} = 0.67 \end{aligned}$$

"z raises the probability that the door is open"

Combining Evidence

Suppose our robot obtains another observation z_2, where the index is the point in time.

Question: How can we integrate this new information?

Formally, we want to estimate $p(\text{open} \mid z_1, z_2)$. Using Bayes formula with background knowledge:

? ?

$$p(\text{open} \mid z_1, z_2) = \frac{p(z_2 \mid \text{open}, z_1)\, p(\text{open} \mid z_1)}{p(z_2 \mid z_1)}$$

Markov Assumption

"If we know the state of the door at time $t = 1$ then the measurement z_1 does not give any further information about z_2."

Formally: "z_1 and z_2 are conditional independent given open." This means:

$$p(z_2 \mid \text{open}, z_1) = p(z_2 \mid \text{open})$$

This is called the ***Markov Assumption.***

Example with Numbers

Assume we have a second sensor:

$$p(z_2 \mid \text{open}) = 0.5 \qquad p(z_2 \mid \neg\text{open}) = 0.6$$

$$p(\text{open} \mid z_1) = \frac{2}{3}$$ (from above)

Then: $p(\text{open} \mid z_1, z_2) =$

$$\frac{p(z_2 \mid \text{open})p(\text{open} \mid z_1)}{p(z_2 \mid \text{open})p(\text{open} \mid z_1) + p(z_2 \mid \neg\text{open})p(\neg\text{open} \mid z_1)}$$

$$= \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{1}{2} \cdot \frac{2}{3} + \frac{3}{5} \cdot \frac{1}{3}} = \frac{5}{8} = 0.625$$

"z_2 **lowers the probability** **that the door is open**"

General Form

Measurements: $z_1, \dots, z_n$

Markov assumption: z_n and $z_1, \dots, z_{n-1}$ are conditionally independent given the state x.

$$p(x \mid z_1, \dots, z_n) = \frac{p(z_n \mid x) p(x \mid z_1, \dots, z_{n-1})}{p(z_n \mid z_1, \dots, z_{n-1})}$$

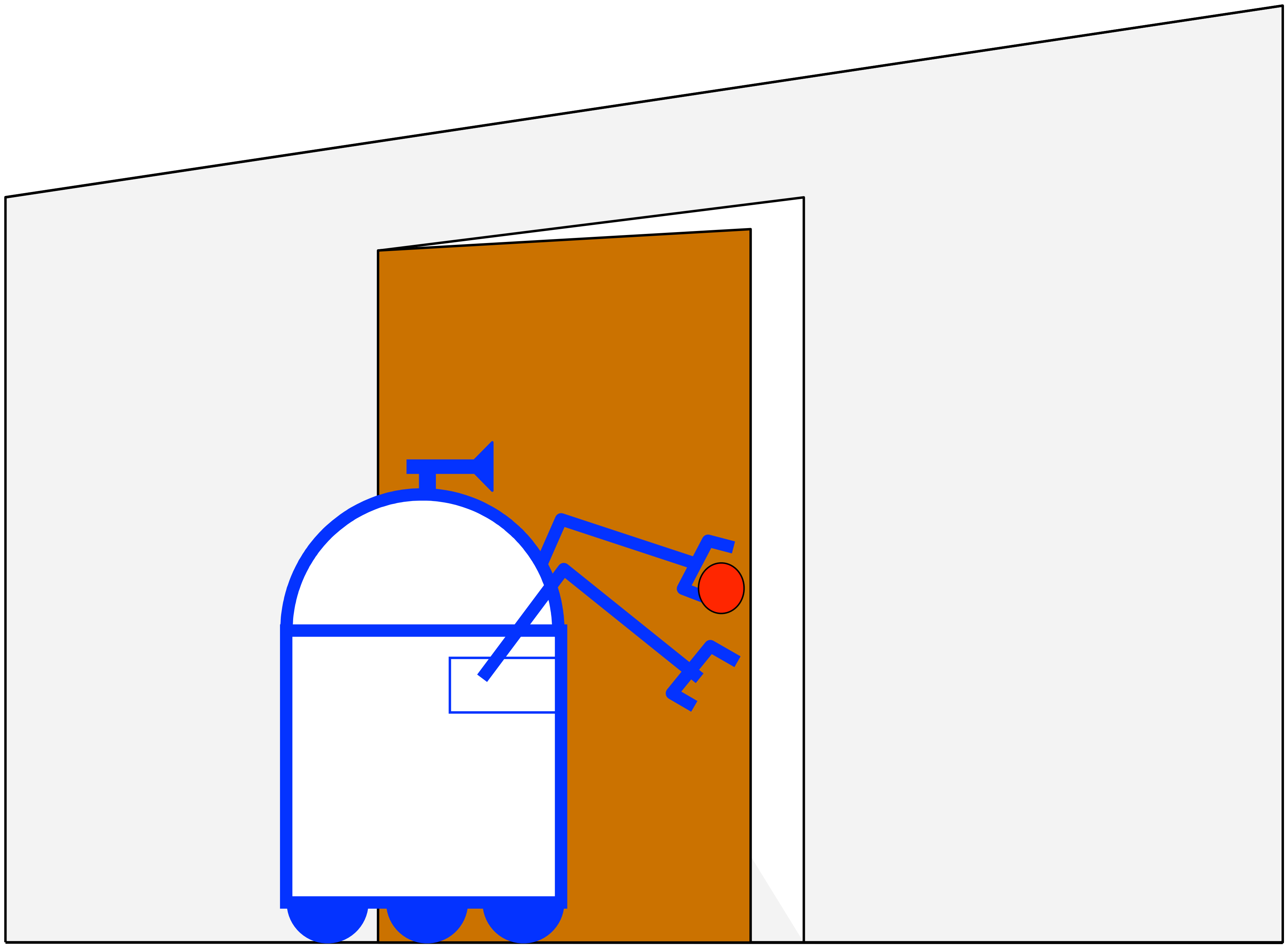
$$= \eta_n \, p(z_n \mid x) p(x \mid z_1, \dots, z_{n-1})$$

Recursion

$$= \prod_{i=1}^{n} \eta_i \, p(z_i \mid x) p(x)$$

Example: Sensing and Acting

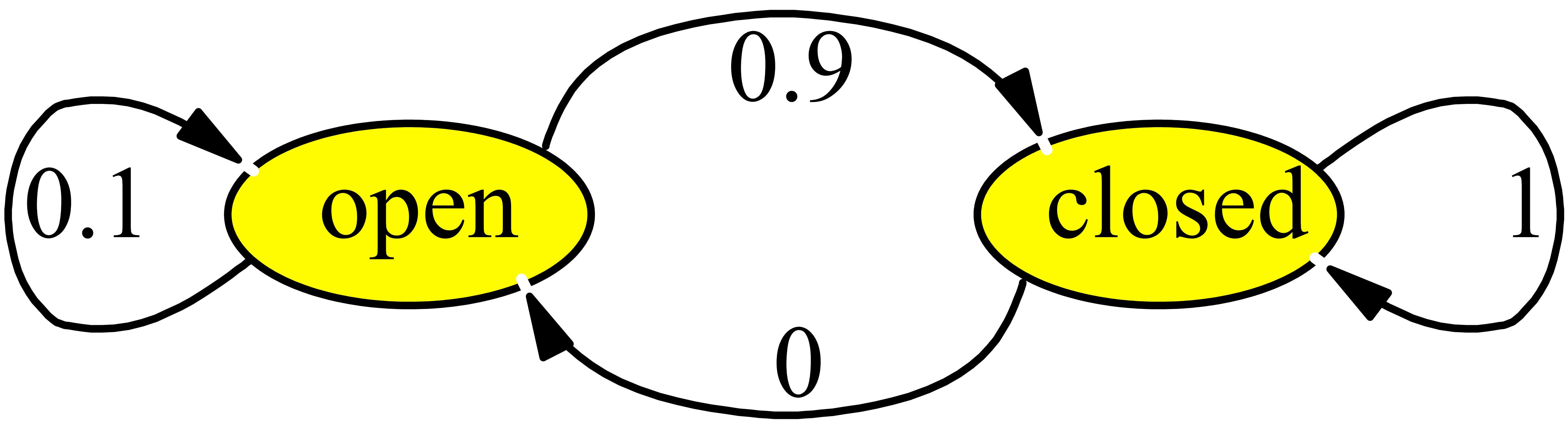
Now the robot ***senses*** the door state and ***acts*** (it opens or closes the door).



State Transitions

The ***outcome*** of an action is modeled as a random variable U where $U = u$ in our case means "state after closing the door".

State transition example:



If the door is open, the action "close door" succeeds in 90% of all cases.

The Outcome of Actions

For a given action u we want to know the probability $p(x \mid u)$. We do this by integrating over all possible previous states x'.

If the state space is discrete:

$$p(x \mid u) = \sum_{x'} p(x \mid u, x') p(x')$$

If the state space is continuous:

$$p(x \mid u) = \int p(x \mid u, x') p(x') dx'$$

Back to the Example

$$\begin{aligned}
p(\text{open} \mid u) &= \sum_{x'} p(\text{open} \mid u, x')p(x') \\
&= p(\text{open} \mid u, \text{open}')p(\text{open}') + \\
&\quad\; p(\text{open} \mid u, \neg\text{open}')p(\neg\text{open}') \\
&= \frac{1}{10} \cdot \frac{5}{8} + 0 \cdot \frac{3}{8} \\
&= \frac{1}{16} = 0.0625
\end{aligned}$$

$$p(\neg\text{open} \mid u) = 1 - p(\text{open} \mid u) = \frac{15}{16} = 0.9375$$

Sensor Update and Action Update

So far, we learned two different ways to update the system state:

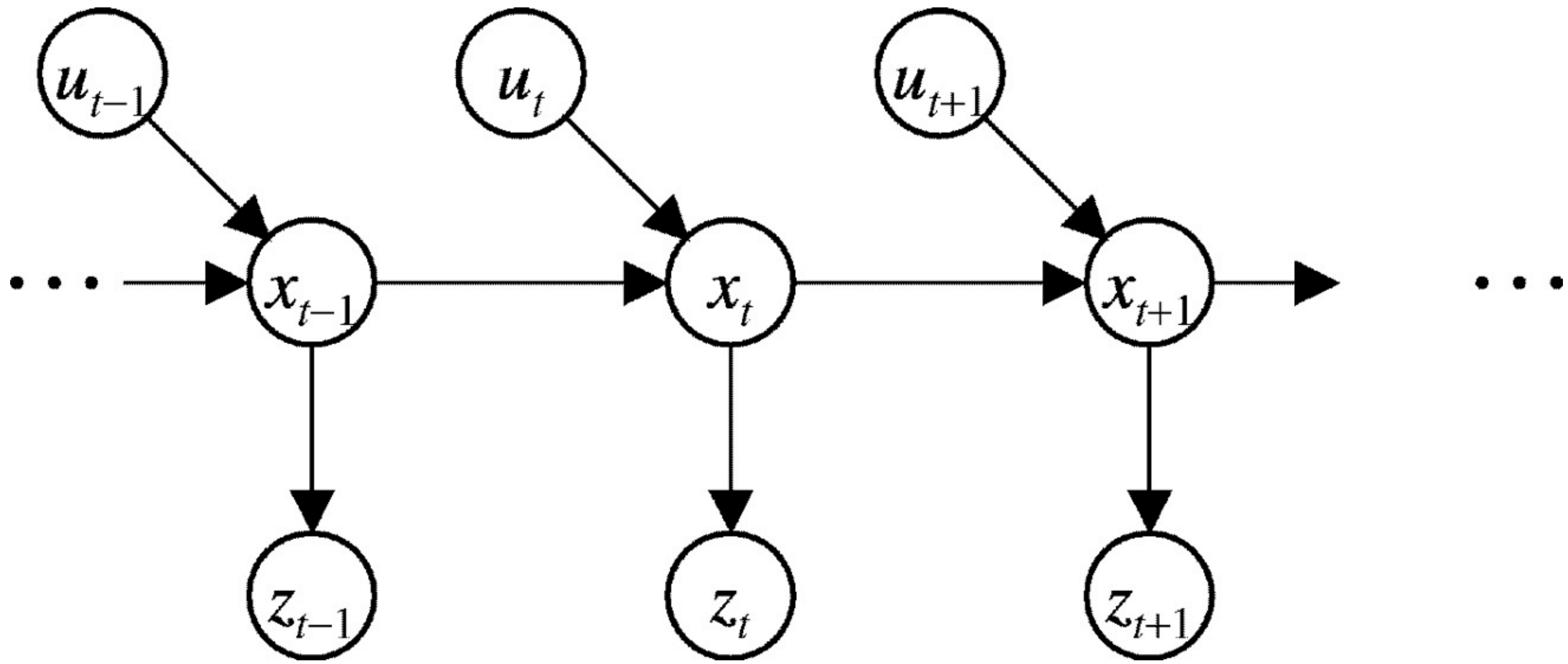
- Sensor update: $p(x \mid z)$
- Action update: $p(x \mid u)$
- Now we want to combine both:

Definition 2.1: Let $D_t = u_1, z_1, \ldots, u_t, z_t$ be a sequence of sensor measurements and actions until time t Then the ***belief*** of the current state x_t is defined as

$$\text{Bel}(x_t) = p(x_t \mid u_1, z_1, \ldots, u_t, z_t)$$

Graphical Representation

We can describe the overall process using a ***Dynamic Bayes Network:***



This incorporates the following Markov assumptions:

$$p(z_t \mid x_{0:t}, u_{1:t}, z_{1:t}) = p(z_t \mid x_t) \quad \text{(measurement)}$$

$$p(x_t \mid x_{0:t-1}, u_{1:t}, z_{1:t}) = p(x_t \mid x_{t-1}, u_t) \quad \text{(state)}$$

The Overall Bayes Filter

$$\mathrm{Bel}(x_t) = p(x_t \mid u_1, z_1, \ldots, u_t, z_t)$$

(Bayes) $$= \eta \; p(z_t \mid x_t, u_1, z_1, \ldots, u_t) p(x_t \mid u_1, z_1, \ldots, u_t)$$

(Markov) $$= \eta \; p(z_t \mid x_t) p(x_t \mid u_1, z_1, \ldots, u_t)$$

(Tot. prob.) $$= \quad \eta \; p(z_t \mid x_t) \int p(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1}) \; p(x_{t-1} \mid u_1, z_1, \ldots, u_t) dx_{t-1}$$

(Markov) $$= \eta \; p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1}) p(x_{t-1} \mid u_1, z_1, \ldots, u_t) dx_{t-1}$$

(Markov) $$= \eta \; p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1}) p(x_{t-1} \mid u_1, z_1, \ldots, z_{t-1}) dx_{t-1}$$

$$= \eta \; p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1}) \mathrm{Bel}(x_{t-1}) dx_{t-1}$$

The Bayes Filter Algorithm

$$\text{Bel}(x_t) = \eta\, p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1}) \text{Bel}(x_{t-1}) dx_{t-1}$$

Algorithm Bayes_filter $(\text{Bel}(x), d)$

1. if d is a sensor measurement z then
2. $\quad \eta = 0$
3. $\quad$ for all x do
4. $\quad\quad \text{Bel}'(x) \leftarrow p(z \mid x)\text{Bel}(x)$
5. $\quad\quad \eta \leftarrow \eta + \text{Bel}'(x)$
6. $\quad$ for all x do $\text{Bel}'(x) \leftarrow \eta^{-1}\text{Bel}'(x)$
7. else if d is an action u then
8. $\quad$ for all x do $\text{Bel}'(x) \leftarrow \int p(x \mid u, x')\text{Bel}(x')dx'$
9. return $\text{Bel}'(x)$

Bayes Filter Variants

$$\text{Bel}(x_t) = \eta\, p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1}) \text{Bel}(x_{t-1}) dx_{t-1}$$

The Bayes filter principle is used in

- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

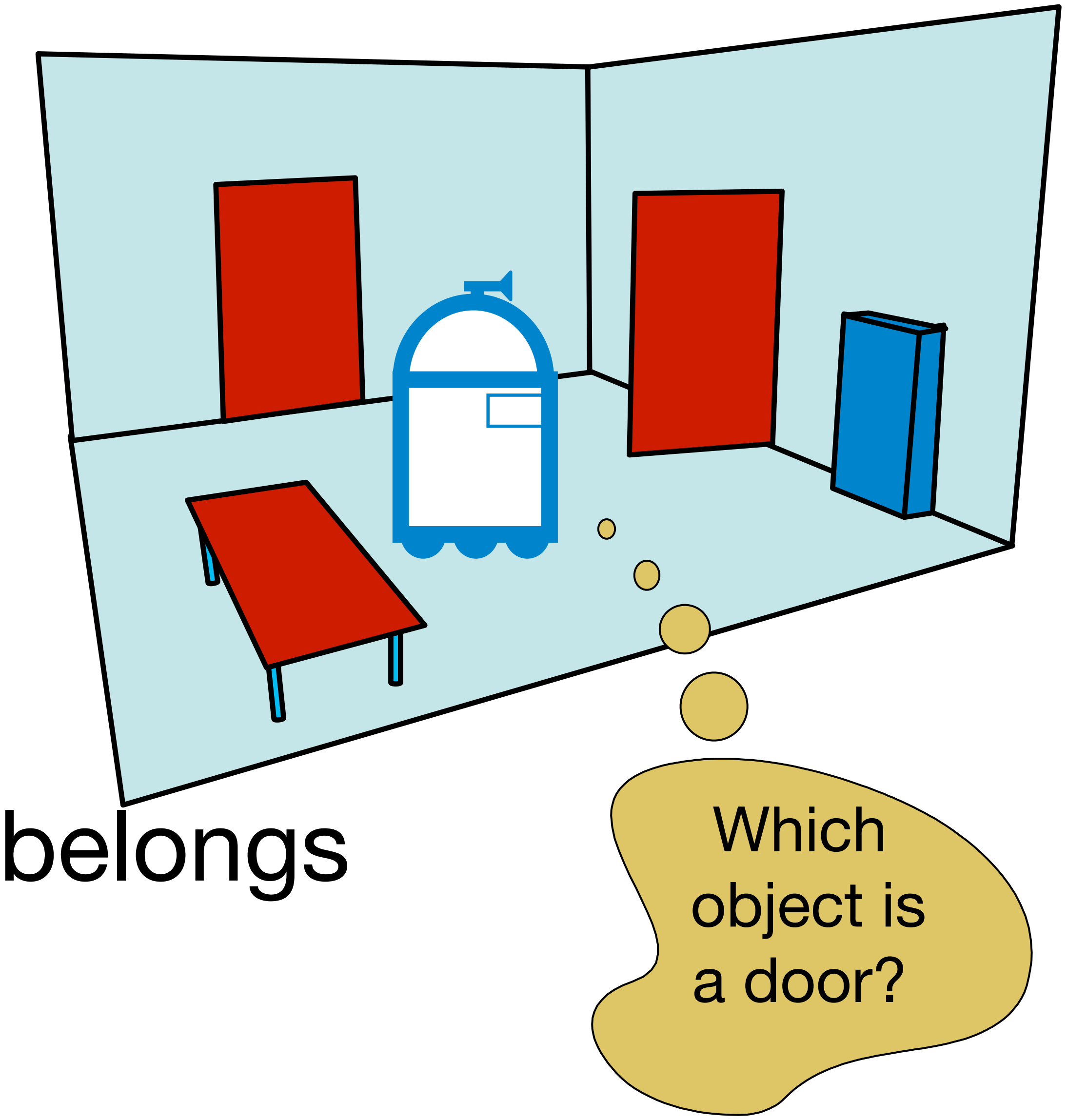
Summary

- ***Probabilistic reasoning*** is necessary to deal with uncertain information, e.g. sensor measurements
- Using ***Bayes rule***, we can do diagnostic reasoning based on causal knowledge
- The outcome of a robot's action can be described by a ***state transition diagram***
- Probabilistic state estimation can be done recursively using the ***Bayes filter*** using a sensor and a motion update
- A graphical representation for the state estimation problem is the ***Dynamic Bayes Network***

2. Introduction to Learning

Motivation

- Most objects in the environment can be classified, e.g. with respect to their size, functionality, dynamic properties, etc.
- Robots need to ***interact*** with the objects (move around, manipulate, inspect, etc.) and with humans
- For all these tasks it is necessary that the robot knows to which ***class*** an object belongs

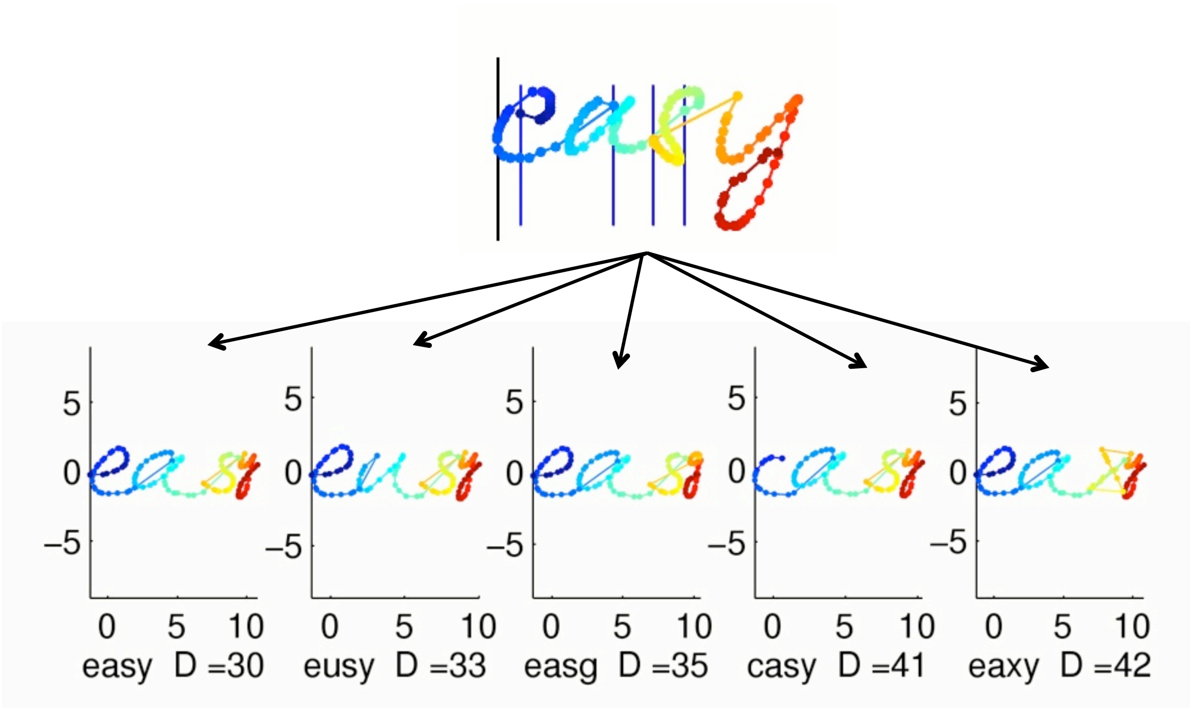


Object Classification Applications

Two major types of applications:

- **Object detection:** For a given test data set find all previously “learned” objects, e.g. pedestrians

- **Object recognition:** Find the particular “kind” of object as it was learned from the training data, e.g. handwritten character recognition



Learning

- A natural way to do object classification is to first **learn** the categories of the objects and then **infer** from the learned data a possible class for a new object.
- The area of **machine learning** deals with the formulization and investigates methods to do the learning automatically.
- Nowadays, machine learning algorithms are more and more used in robotics and computer vision

Mathematical Formulation

Suppose we are given a set $\mathcal{X}$ of objects and a set $\mathcal{Y}$ of object categories (classes). In the learning task we search for a mapping $\varphi : \mathcal{X} \rightarrow \mathcal{Y}$ such that **similar** elements in $\mathcal{X}$ are mapped to **similar** elements in $\mathcal{Y}$.

Examples:

- Object classification: chairs, tables, etc.
- Optical character recognition
- Speech recognition

Important problem: Measure of similarity!

Categories of Learning

Learning

- **Unsupervised Learning**: clustering, density estimation
- **Supervised Learning**: **learning** from a training data set, **inference** on the test data
- **Reinforcement Learning**: no supervision, but a **reward function**

Supervised Learning

- **Discriminant Function**: no prob. formulation, learns a **function** from objects $\mathcal{X}$ to labels $\mathcal{Y}$.
- **Discriminative Model**: estimates the **posterior** $p(y_k \mid \mathbf{x})$ for each class
- **Generative Model**: est. the **likelihoods** $p(\mathbf{x} \mid y_k)$ and use Bayes rule for the post.

Categories of Learning

Learning

Unsupervised Learning	**Supervised Learning**	Reinforcement Learning
clustering, density estimation	**learning** from a training data set, **inference** on the test data	no supervision, but a reward function

Supervised Learning is the main topic of this lecture!

Methods used in Computer Vision include:

- Regression
- Conditional Random Fields
- Boosting
- Support Vector Machines
- Gaussian Processes
- Hidden Markov Models

Categories of Learning

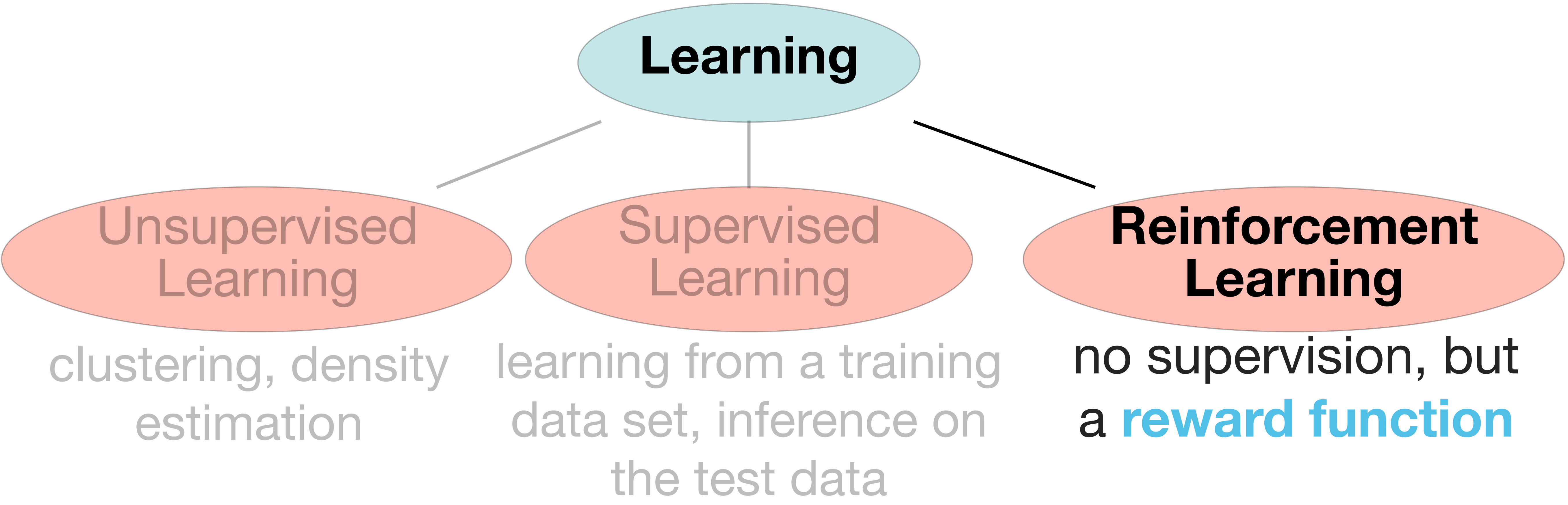
Learning

- **Unsupervised Learning**: clustering, density estimation
- Supervised Learning: learning from a training data set, inference on the test data
- Reinforcement Learning: no supervision, but a reward function

Most Unsupervised Learning methods are based on Clustering.

➡Will be handled at the end of this semester

Categories of Learning



Reinforcement Learning requires an ***action***

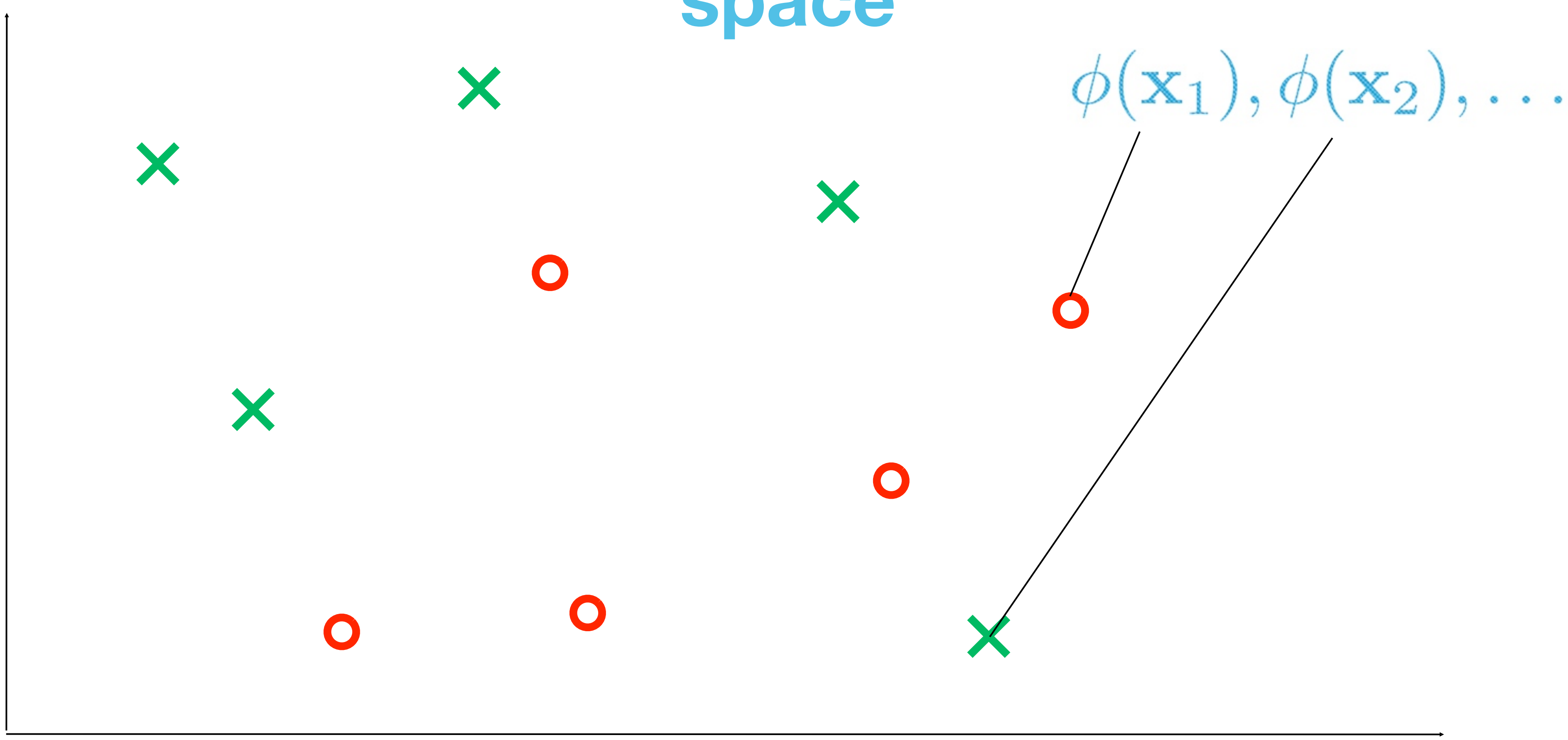
- the reward defines the quality of an action
- mostly used in robotics (e.g. manipulation)
- can be dangerous, actions need to be “tried out”
- not handled in this course

Generative Model: Example

Nearest-neighbor classification:

- Given: data points $(\mathbf{x}_1, t_1), (\mathbf{x}_2, t_2), \ldots$
- Rule: Each new data point is assigned to the class of its nearest neighbor in feature space

1. Training instances in feature space

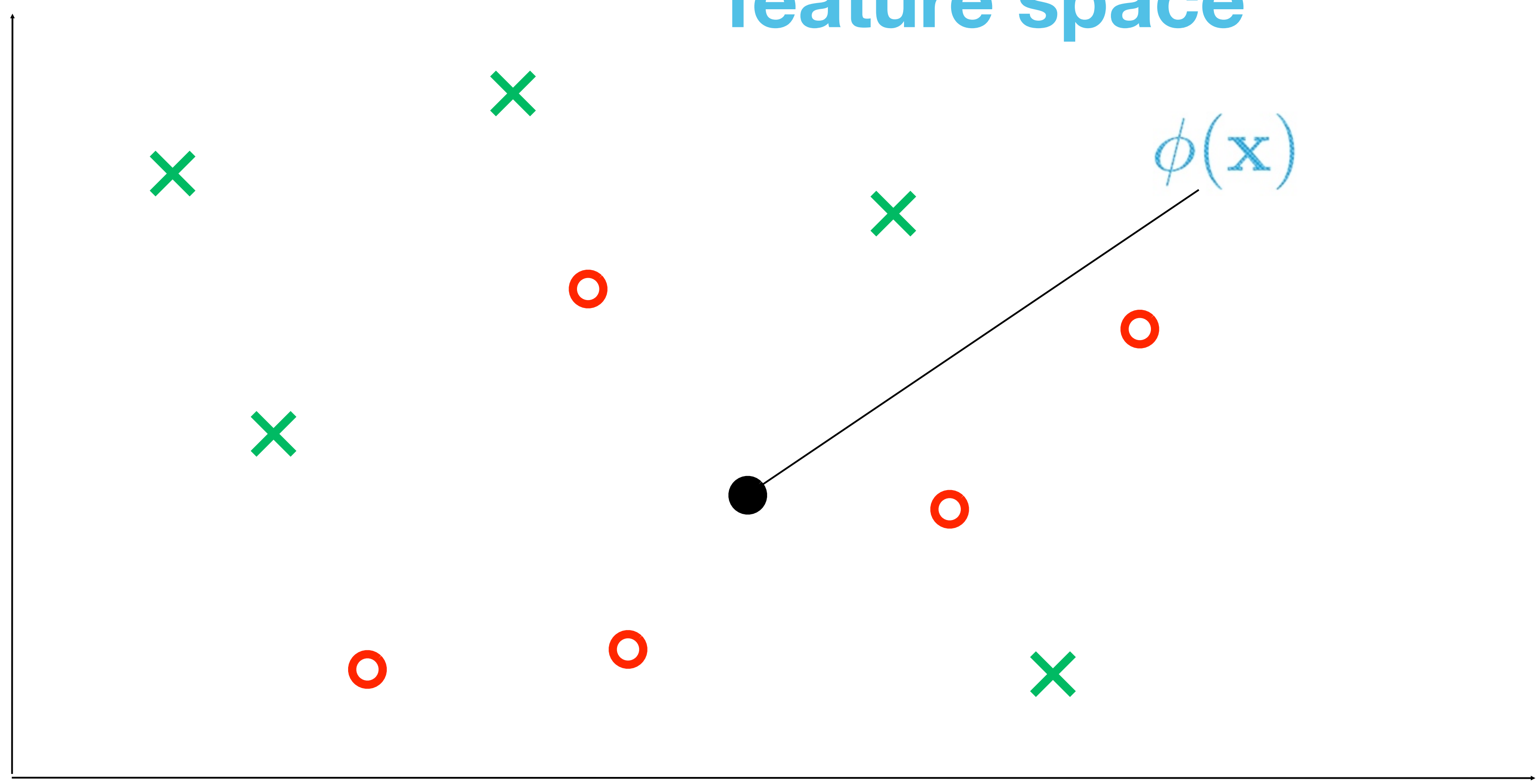


Generative Model: Example

Nearest-neighbor classification:

- Given: data points $(\mathbf{x}_1, t_1), (\mathbf{x}_2, t_2), \ldots$
- Rule: Each new data point is assigned to the class of its nearest neighbor in feature space

2. Map new data point into feature space

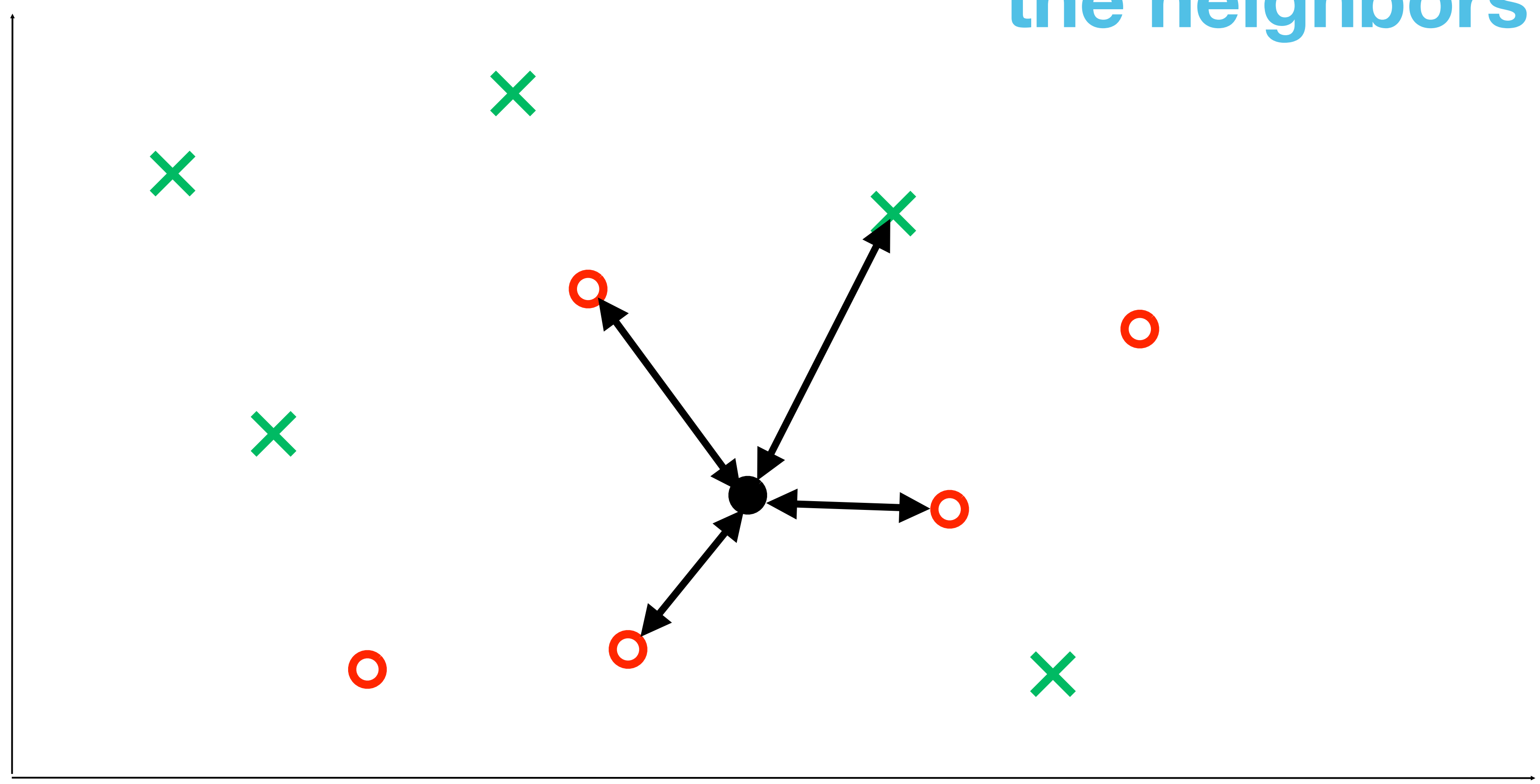


Generative Model: Example

Nearest-neighbor classification:

- Given: data points $(\mathbf{x}_1, t_1), (\mathbf{x}_2, t_2), \ldots$
- Rule: Each new data point is assigned to the class of its nearest neighbor in feature space

3. Compute the distances to the neighbors

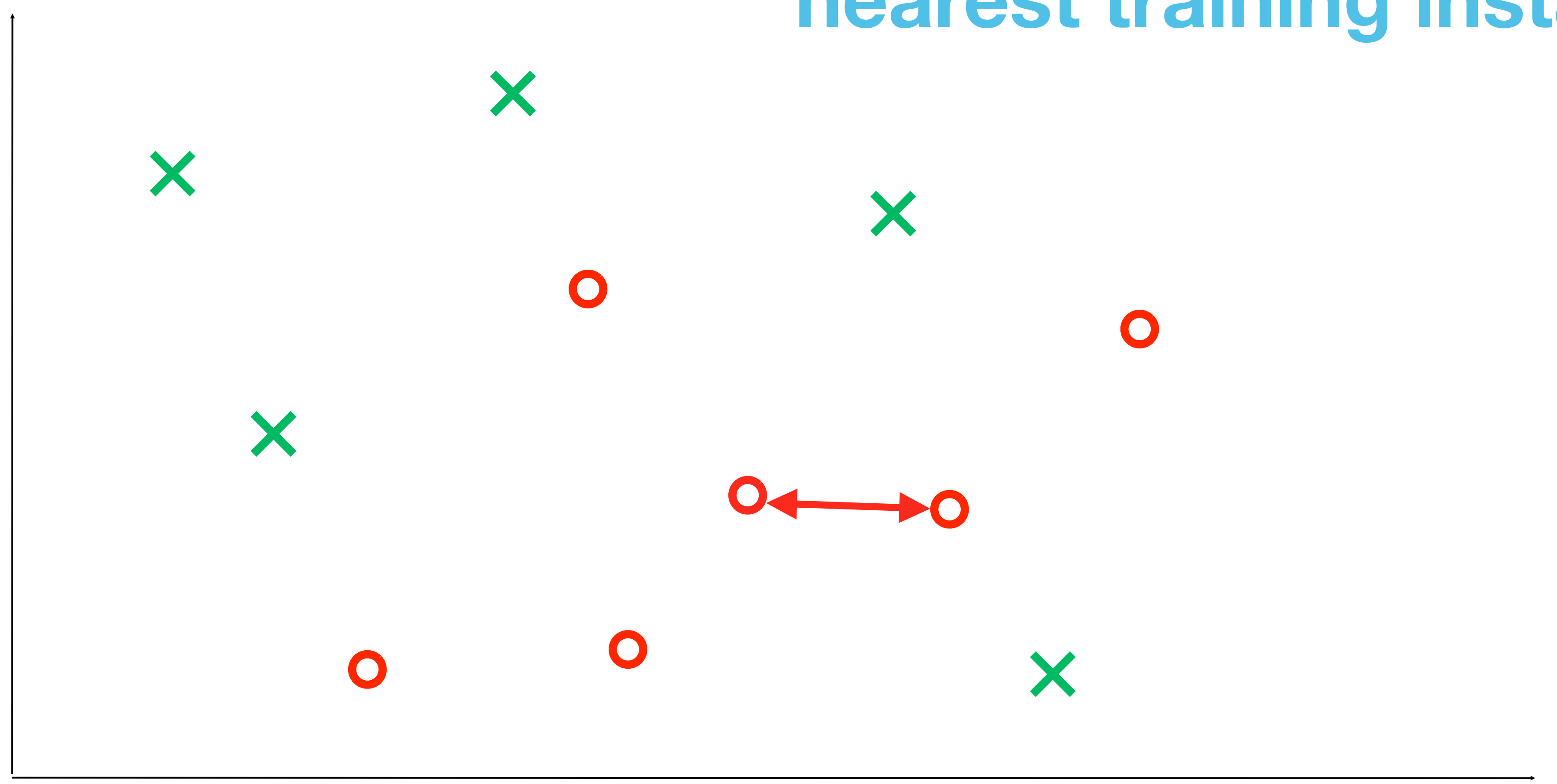


Generative Model: Example

Nearest-neighbor classification:

- Given: data points $(\mathbf{x}_1, t_1), (\mathbf{x}_2, t_2), \ldots$
- Rule: Each new data point is assigned to the class of its nearest neighbor in feature space

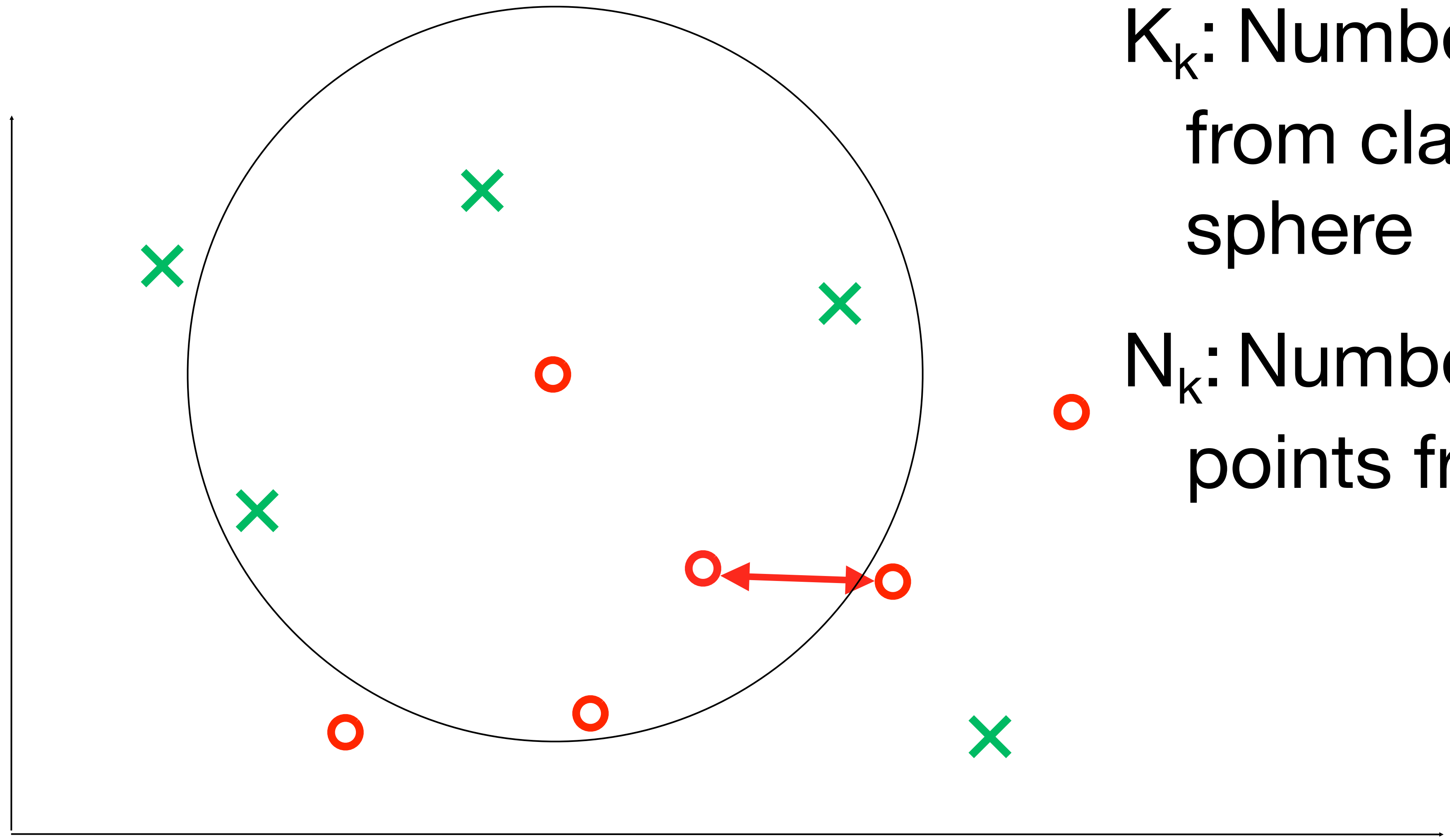
4. Assign the label of the nearest training instance



Generative Model: Example

Nearest-neighbor classification:

- General case: K nearest neighbors
- We consider a sphere around each training instance that has a fixed volume V.



K_k: Number of points from class k inside sphere

N_k: Number of all points from class k

Generative Model: Example

Nearest-neighbor classification:

- General case: K nearest neighbors
- We consider a sphere around each training instance that has a fixed volume V.
- With this we can estimate:

$$p(\mathbf{x} \mid y = k) = \frac{K_k}{N_k V}$$

"likelihood"

- and likewise:

$$p(\mathbf{x}) = \frac{K}{NV}$$

points in sphere (K)

all points (N)

"uncond. prob."

- using Bayes rule.

$$p(y = k \mid \mathbf{x}) = \frac{p(\mathbf{x} \mid y = k)p(y = k)}{p(\mathbf{x})} = \frac{K_k}{K}$$

"posterior"

Generative Model: Example

Nearest-neighbor classification:

- General case: K nearest neighbors

$$p(y = k \mid \mathbf{x}) = \frac{p(\mathbf{x} \mid y = k)p(y = k)}{p(\mathbf{x})} = \frac{K_k}{K}$$

- To classify the new data point $\mathbf{x}$ we compute the posterior for each class k = 1,2,… and assign the label that maximizes the posterior.

$$t := \arg\max_k p(y = k \mid \mathbf{x})$$

Summary

- Learning is a two-step process consisting in a ***training*** and an ***inference*** step
- Learning is useful to extract ***semantic*** information, e.g. about the objects in an environment
- There are three main categories of learning: ***unsupervised***, ***supervised*** and ***reinforcement*** learning
- Supervised learning can be split into ***discriminant function***, ***discriminant model***, and ***generative model*** learning
- An example for a generative model is ***nearest neighbor classification***