TU München
Fakultät für Informatik
Dr. Rudolph Triebel
Jan Stühmer

Machine Learning for Robotics and Computer Vision
Summer term 2013

Solution Sheet 2
Topic: Regression
May 24th, 2013

Exercise 1:

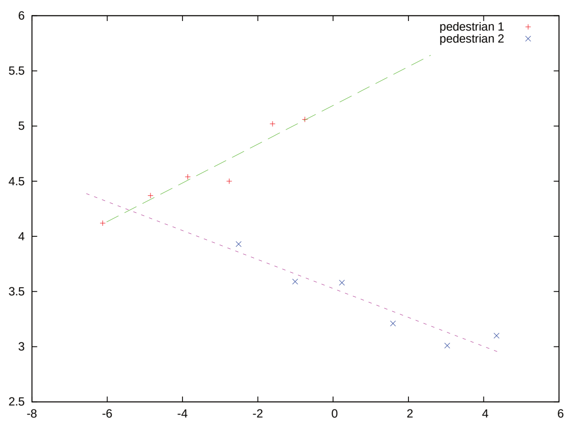

Abbildung 1: Scan data from two pedestrians. The lines are the result of the linear regressions in x and y-direction.

a) See figure.

b) The task is to predict the motion of the pedestrians. We do this using polynomial regression. The functions that we learn are dependant on time. We have to find two functions, one of the x-coordiante and one for the y-coordinates. The regression is done with the matrix Φ and vectors $\mathbf{t}_i$:

$$\Phi = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{pmatrix} \quad \mathbf{t}_{x1} = \begin{pmatrix} -6.12 \\ -4.85 \\ -3.86 \\ -2.76 \\ -1.61 \\ -0.751 \end{pmatrix} \mathbf{t}_{y1} = \begin{pmatrix} 4.12 \\ 4.37 \\ 4.54 \\ 4.5 \\ 5.02 \\ 5.06 \end{pmatrix} \mathbf{t}_{x2} = \begin{pmatrix} 4.34 \\ 3.03 \\ 1.59 \\ 0.233 \\ -1.01 \\ -2.51 \end{pmatrix} \mathbf{t}_{y2} = \begin{pmatrix} 3.1 \\ 3.01 \\ 3.21 \\ 3.58 \\ 3.59 \\ 3.93 \end{pmatrix}$$

The second column of Φ are the timestamps at which the measurements have been taken. In this first case, we assume constant velocity, i.e. we don't have acceleration and the motion equation has only two unkowns w_0 and w_1, i.e. for the case of the x-coordinates we have

$$x(\tau) = w_0 + w_1\tau, \qquad \mathbf{w}_x = (w_0, w_1)^T$$

where $\tau = 0, 1, \ldots$ is the time stamp. Thus, Φ has two cloumns.

The pseudoinverse of Φ is

$$\Phi^\dagger = \begin{pmatrix} 0.524 & 0.381 & 0.238 & 0.095 & -0.048 & -0.190 \\ -0.143 & -0.086 & -0.029 & 0.029 & 0.086 & 0.143 \end{pmatrix}$$

With this we compute $\mathbf{w}_i = \Phi^\dagger \mathbf{t}_i$:

$$\mathbf{w}_{x1} = \Phi^\dagger \mathbf{t}_{x1} = \begin{pmatrix} -6.016 \\ 1.076 \end{pmatrix} \quad \mathbf{w}_{y1} = \begin{pmatrix} 4.130 \\ 0.189 \end{pmatrix} \quad \mathbf{w}_{x2} = \begin{pmatrix} 4.355 \\ -1.364 \end{pmatrix} \quad \mathbf{w}_{y2} = \begin{pmatrix} 2.956 \\ 0.179 \end{pmatrix}$$

Using that we can compute the time to reach $x = 3$ and $x = -5$ respectively:

$$\begin{aligned} \tau_1 &= \frac{3 - w_{x1}^0}{w_{x1}^1} = 8.378s && (1) \\ \tau_2 &= \frac{-5 - w_{x2}^0}{w_{x2}^1} = 6.860s && (2) \\ & && (3) \end{aligned}$$

To compute the speeds we need $\mathbf{v}_1 = (1.076, 0.189)^T$ and $\mathbf{v}_2 = (-1.364, 0.179)^T$ The speeds are $\|\mathbf{v}_1\| = 1.093$ and $\|\mathbf{v}_1\| = 1.375$. The residual errors are defined as

$$\begin{aligned} r_{x1} &= \|\Phi \mathbf{w}_{x1} - \mathbf{t}_{x1}\| = 0.208 && (4) \\ r_{y1} &= \|\Phi \mathbf{w}_{y1} - \mathbf{t}_{y1}\| = 0.246 && (5) \\ r_{x2} &= \|\Phi \mathbf{w}_{x2} - \mathbf{t}_{x2}\| = 0.120 && (6) \\ r_{y2} &= \|\Phi \mathbf{w}_{y2} - \mathbf{t}_{y2}\| = 0.260 && (7) \\ & && (8) \end{aligned}$$

c) Now we have a quadratic motion equation:

$$x(\tau) = w_0 + w_1\tau + w_2\tau^2, \qquad \mathbf{w}_x = (w_0, w_1, w_2)^T,$$

where w_1 is velocity and w_2 is (half the) acceleration. This means we have to estimate 3 function parameters. Thus, the matrix Φ has one more column, i.e.

$$\Phi = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{pmatrix}$$

Again, we compute the pseudoinverse and multiply it with the vectors $\mathbf{t}_i$. We obtain:

$$\mathbf{w}_{x1} = \begin{pmatrix} -6.01 \\ 1.2 \\ -0.03 \end{pmatrix} \quad \mathbf{w}_{y1} = \begin{pmatrix} 4.15 \\ 0.158 \\ 0.006 \end{pmatrix} \quad \mathbf{w}_{x2} = \begin{pmatrix} 4.345 \\ -1.349 \\ -0.003 \end{pmatrix} \quad \mathbf{w}_{y2} = \begin{pmatrix} 3.039 \\ 0.055 \\ 0.025 \end{pmatrix}$$

The residual errors are now

$$
\begin{aligned}
r_{x1} &= 0.1397 && (9)\\
r_{y1} &= 0.2434 && (10)\\
r_{x2} &= 0.1183 && (11)\\
r_{y2} &= 0.2114 && (12)\\
& && (13)
\end{aligned}
$$

Exercise 2: Programming

See Code.