Variational Methods for Computer Vision: Exercise Sheet 5

Exercise: 28 November 2013

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Calculate the Euler-Lagrange equation of the following form of the internal energy of the Snakes segmentation model:

$$E(C) = \frac{1}{2}\int_0^1 \alpha(s)|C(s)'|^2\, ds + \frac{1}{2}\int_0^1 \beta(s)|C(s)''|^2\, ds$$

   Note that the parameters β and α depend on s.

2. Let $c : [0;1] \rightarrow \mathbb{R}^2$ be a parametrized curve on the image plane. Consider the following functional:

$$L(C) := \int_0^1 |C'(s)|\, ds$$

   (a) Calculate the Euler-Lagrange Equation of $L(C)$.

   (b) Prove that the measure $L(c)$ is independent of a parametrization m of the curve C hence

$$\int_0^1 |C'(m(s))|\, ds = \int_0^1 |C'(s)|\, ds$$

   where $m : [0,1] \rightarrow [0,1]$ denotes a strictly increasing function such that $m(0) = 0$ and $m(1) = 1$.

   (c) Prove that the following equality:

$$\int_0^1 |C'(s)|\, ds = \int_0^1 |C'(s)|^2\, ds$$

   if $|C'(s)|$ is constant.

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Continue with the practical exercise of last weeks exercise sheet.