# Variational Methods for Computer Vision: Exercise Sheet 10

Exercise: 23 January 2014

## Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. The general formulation of the total variation applies also to discontinuous functions $u : \Omega \to \mathbb{R}$ and has the following form:

$$E(u) = \sup_{\varphi \in \mathcal{K}} \left\{ \int_{\Omega} u \text{div}\, \varphi \, dx \right\}$$

s.t.

$$\mathcal{K} := \left\{ \varphi \in C_c^1(\Omega; \mathbb{R}^2) \;\middle|\; |\varphi(x)| \leq 1 \quad \forall x \in \Omega \right\}$$

Prove that the above formulation is a convex functional.

2. Let $I : \Omega \times \mathbb{R}_+ \to \mathbb{R}$ be an image sequence and $v =: (v_1, v_2)^T : \Omega \to \mathbb{R}^2$ be a vector field. Compute the Euler-Lagrange equation of the following functional:

$$E(v) = \int_{\Omega} \big( \langle \nabla I(x), v \rangle + I_t(x) \big)^2 \, dx + \alpha \int_{\Omega} |\nabla v_1(x)|^2 + |\nabla v_2(x)|^2 \, dx$$

## Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Implement the optical flow method of Lukas and Kanade and apply it on the images in the image sequence contained in the archive `vmcv_ex10.zip`.
2. Generate an image which colour codes all pixels moving to left in white and pixels moving to the right in black.