Variational Methods for Computer Vision: Solution Sheet 7

Exercise: 19 December 2013

Part I: Theory

1.

$$
\begin{aligned}
E'(t) &= \int_0^1 \frac{\mathrm{d}}{\mathrm{d}t}\left(g(C(t,s))\left|\frac{\partial C}{\partial s}(t,s)\right|\right)\mathrm{d}s \\
&= \int_0^1 \nabla g \frac{\partial C}{\partial t}\left|\frac{\partial C}{\partial s}(t,s)\right| + g(C)\cdot\frac{\frac{\partial C}{\partial s}}{\left|\frac{\partial C}{\partial s}\right|}\cdot\frac{\partial C}{\partial t\partial s}\mathrm{d}s \\
&= \int_0^1 \nabla g \frac{\partial C}{\partial t}\left|\frac{\partial C}{\partial s}(t,s)\right| - \left(\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\frac{\partial C}{\partial s}}{\left|\frac{\partial C}{\partial s}\right|}\right)g(C) - \nabla g\cdot\frac{\partial C}{\partial s}\cdot\frac{\frac{\partial C}{\partial s}}{\left|\frac{\partial C}{\partial s}\right|}\right)\frac{\partial C}{\partial s}\mathrm{d}s \\
&= \int_0^1 \left(\nabla g - \left(\frac{1}{\left|\frac{\partial C}{\partial s}\right|}\frac{\partial}{\partial s}\left(\frac{\frac{\partial C}{\partial s}}{\left|\frac{\partial C}{\partial s}\right|}\right)\right)\cdot g(C) - \nabla g\cdot\left(\frac{\frac{\partial C}{\partial s}}{\left|\frac{\partial C}{\partial s}\right|}\right)\cdot\left(\frac{\frac{\partial C}{\partial s}}{\left|\frac{\partial C}{\partial s}\right|}\right)\right)\frac{\partial C}{\partial t}\left|\frac{\partial C}{\partial s}\right|\mathrm{d}s \\
&= \int_0^1 \left(\nabla g - \kappa n g - (\nabla g\cdot T)\,T\right)\frac{\partial C}{\partial t}\left|\frac{\partial C}{\partial s}\right|\mathrm{d}s \qquad (*) \\
&= \int_0^1 \left((\nabla g\cdot n)\,n - \kappa n g\right)\cdot\frac{\partial C}{\partial t}\left|\frac{\partial C}{\partial s}\right|\mathrm{d}s \\
&\Rightarrow \frac{\partial C}{\partial t} = (\kappa g - (\nabla g\cdot n))n
\end{aligned}
$$

* Using $\nabla g = (\nabla g\cdot n)n + (\nabla g\cdot T)T$