Variational Methods for Computer Vision: Exercise Sheet 3

Exercise: 27 November 2014

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let $L : X \to Y$ be a linear operator and X, Y be finite dimensional vector spaces with dim $X = n$ and dim $Y = m$. Let $\{e_1, ..., e_n\}$ and $\{\tilde{e}_1, ..., \tilde{e}_m\}$ be the bases for X and respectively for Y. Show that the operator L can be represented by an $m \times n$ matrix M, hence:

$$L(u) = Mu, \qquad \forall u \in X.$$

2. Calculate the Euler-Lagrange equation of the following energy functional

$$E(u) = \int_\Omega \frac{\lambda}{2} \left((k * u)(x) - f(x)\right)^2 + |\nabla u(x)| \, \mathrm{dx},$$

where $\Omega \subset \mathbb{R}^2$ represents the image domain, $u : \Omega \to \mathbb{R}$ denotes the optimization variable, $f : \Omega \to \mathbb{R}$ stands for the input image and $k : \Omega \to \mathbb{R}$ denotes a convolution kernel (not necessarily symmetrical).

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Finish the exercises from the previous sheet.

2. In the first theoretical exercise we showed that every linear operation on a finite dimensional space can be represented as a matrix vector multiplication. Since the convolution operation is linear its possible to represent it as such. Write a script that implements convolution with a Gaussian kernel as a sparse matrix-vector multiplication.

Matlab-Tutorials:
`http://www.math.utah.edu/lab/ms/matlab/matlab.html`
`http://www.math.ufl.edu/help/matlab-tutorial/`
`http://www.glue.umd.edu/~nsw/ench250/matlab.htm`