Variational Methods for Computer Vision: Exercise Sheet 10

Exercise: January 22, 2015

Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. The Chan-Vese functional $E(\phi)$ from last exercise sheet has been reformulated by Chan, Esodoglu and Nikolova by associating $u \equiv H(\phi)$ where $u : \Omega \to [1; 0]$. The resulting functional can be written as follows:

$$E(u) = \int_\Omega f_1(x)u + f_2(x)(1-u) + \nu|\nabla u| \text{ dx} \tag{1}$$

   (a) Prove that $E(u)$ is a convex functional.

   (b) Prove that the family of functions $U := \{u : \Omega \to [0; 1]\}$ is a convex function space. Hence that for all pairs $u_1, u_2 \in U$ every linear combination is again in U:

$$\lambda_1 u_1 + \lambda_2 u_2 \in U \qquad \forall \lambda_1, \lambda_2 > 0, \text{ s.t } \lambda_1 + \lambda_2 = 1$$

   (c) The projection $f_U \in U$ of a given function $f : \Omega \to \mathbb{R}$ onto the convex function space U can be written as the minimizer of the following functional:

$$f_U := \underset{u \in U}{\operatorname{argmin}} \left(\int_\Omega (f(x) - u(x))^2 \text{ dx} \right)$$

   show that :

$$f_U(x) = \begin{cases} 1 & \text{if} \quad f(x) > 1 \\ 0 & \text{if} \quad f(x) < 0 \\ f(x) & \text{else.} \end{cases}$$

   (d) Prove that the Euler-Lagrange equation of E(u) can be written as follows:

$$\frac{dE}{du} = \left[f_1 - f_2 - \nu \operatorname{div} \left(\frac{\nabla u}{|\nabla u|} \right) \right] = 0$$

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Implement the minimization of the Chan-Esodoglu-Nikolova functional and make sure the optimization stays in the constrained space of functions U from the theoretical exercise by doing a re-projection by clipping (as in exercise 1c).

2. Test your implementation on the image `image.png` from last exercise sheet by initializing the the algorithm with a circle of radius R in the center of the image.

3. After obtaining the global minimizer visualize the segmentation result by thresholding the resulting function i.e by using the command `imagesc(u<0.5)`.Test also other thresholds than 0.5.