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# Variational Methods for Computer Vision: Exercise Sheet 10

Exercise: January 22, 2015

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## Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. The Chan-Vese functional  $E(\phi)$  from last exercise sheet has been reformulated by Chan, Esdoğlu and Nikolova by associating  $u \equiv H(\phi)$  where  $u : \Omega \rightarrow [0; 1]$ . The resulting functional can be written as follows:

$$E(u) = \int_{\Omega} f_1(x)u + f_2(x)(1 - u) + \nu|\nabla u| \, dx \quad (1)$$

- (a) Prove that  $E(u)$  is a convex functional.
- (b) Prove that the family of functions  $U := \{u : \Omega \rightarrow [0; 1]\}$  is a convex function space. Hence that for all pairs  $u_1, u_2 \in U$  every linear combination is again in  $U$ :

$$\lambda_1 u_1 + \lambda_2 u_2 \in U \quad \forall \lambda_1, \lambda_2 > 0, \text{ s.t. } \lambda_1 + \lambda_2 = 1$$

- (c) The projection  $f_U \in U$  of a given function  $f : \Omega \rightarrow \mathbb{R}$  onto the convex function space  $U$  can be written as the minimizer of the following functional:

$$f_U := \operatorname{argmin}_{u \in U} \left( \int_{\Omega} (f(x) - u(x))^2 \, dx \right)$$

show that :

$$f_U(x) = \begin{cases} 1 & \text{if } f(x) > 1 \\ 0 & \text{if } f(x) < 1 \\ u(x) & \text{else.} \end{cases}$$

- (d) Prove that the Euler-Lagrange equation of  $E(u)$  can be written as follows:

$$\frac{dE}{du} = \left[ f_1 - f_2 - \nu \operatorname{div} \left( \frac{\nabla u}{|\nabla u|} \right) \right] = 0$$

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## Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Implement the minimization of the Chan-Esodoglu-Nikolova functional and make sure the optimization stays in the constrained space of functions  $U$  from the theoretical exercise by doing a re-projection by clipping (as in exercise 1c).
2. Test your implementation on the image `image.png` from last exercise sheet by initializing the the algorithm with a circle of radius  $R$  in the center of the image.
3. After obtaining the global minimizer visualize the segmentation result by thresholding the resulting function i.e by using the command `imagesc(u<0.5)`. Test also other thresholds than 0.5.