

# Seminar: Selected Topics in Variational Image Processing

Oriental meeting

WS 2014/2015

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Thomas Möllenhoff,  
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Seminar: Selected  
Topics in Variational  
Image Processing

Michael Möller,  
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# Objectives of the seminar

Seminar: Selected  
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## What is happening here?

- We briefly present 15 seminar topics.



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- The last seminar is the week before Christmas.
- You have to write a 5-7 page report about your topic which is due on Jan. 7th, 2015.



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# What you will learn in this seminar

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## Goals

- Get an impression of recent advances in variational image processing in various applications.
- Learn how to study a recent research paper and get a deep understanding of one particular topic.
- Write a scientific report.
- Practice giving scientific talks.

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# Requirements, or “is this something for me?”

## Necessary

- Good background and interest in mathematics.
- Working knowledge about basic linear algebra and multivariable calculus in finite dimensions.

## Recommended

- Computer Vision fundamentals from any basic course.
- Having heard about variational methods.



# Important Dates



- First meeting: Today (02.07.2014)
- Registration: starting from **04.07.**
- We assign the topics and talks and will write you an email.
- Weekly presentations starting on **Wednesday** Oct. 29th, **14:15-16:00.**
- Read and discuss your assigned topic with your supervisor **early.**
- Deliver and discuss your slides **one week before your presentation.**
- Hand in your report until **Jan. 7th, 2015.**



Please do not work on your topic completely alone!

- Meet your supervisor at least **twice**.
- We recommend: Discuss your topic with your supervisor **one month before your talk**.
- We require: Deliver and discuss your slides **one week before your presentation**.



- The report should contain an **overview** and the **main contributions** of your assignment.
- Length: **5-7 pages**.
- Language: **English** or **German**.
- Write your report with **Latex** – a template will be available on the course web page.
- Send a **PDF via email** to your supervisor.
- Hand in your report until **Jan. 7th, 2015**.



- **35min talk** with **10min discussion** afterwards.
- Don't put too much information on one slide – 1-2 minutes per slide, i.e. not more than 35 slides!
- Language: **English**.
- You are free to choose the presentation software but need to export to PDF for discussion with your supervisor.

# Evaluation Criteria



You will be evaluated based on the following criteria:

- Gained expertise in the topic.
- Quality of your talk.
- Quality of your report.
- Active participation in the seminar is expected (questions + comments after the talks).
  
- **Attendance of each seminar is mandatory!**  
In case of absence: medical certificate.



### Variational Methods for Computer Vision

- Structured introduction to variational methods.
- Learn variational modelling for several applications.



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### Recent advances in computer vision: Numerical Methods for Variational Image Analysis

- After the modeling one typically ends up with  $\hat{u} = \arg \min_u E(u)$
- This lecture is about the theory and implementation of numerical optimization methods for actually solving the above problem.



# Overview of available topics

# An Introduction to Total Variation for Image Analysis

- Introduction to image processing and inverse problems.

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## An Introduction to Total Variation for Image Analysis

- Introduction to image processing and inverse problems.
- Recover unknown image  $u$  from observed image  $f$  with noise  $\eta$ .

$$f = Au + \eta$$



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$$f = Au + \eta$$

- Highly *ill-posed* problem  $\rightarrow$  regularization!
- Formulation as an energy minimization problem:

$$\operatorname{argmin}_u \underbrace{\|Au - f\|^2}_{\text{Dataterm}} + \underbrace{J(u)}_{\text{Regularizer}}$$



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- Total Variation (TV) is one of the most versatile regularizers with many interesting properties.



(a) Original image



(b) Degraded image



(c) Wiener filter



(d) TV-deblurring



## A Convex Formulation of Continuous Multi-Label Problems

- Many optimization problems in computer vision are *nonconvex* and even NP-hard.
- Direct / naive minimization usually leads to poor local optima.

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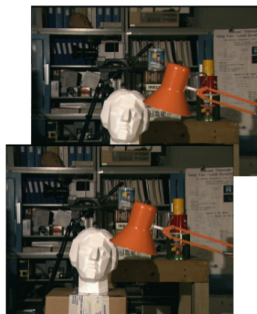
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## A Convex Formulation of Continuous Multi-Label Problems

- Many optimization problems in computer vision are *nonconvex* and even NP-hard.
- Direct / naive minimization usually leads to poor local optima.
- Replace initial nonconvex problem by a equivalent higher dimensional convex problem:

$$\operatorname{argmin}_{u:\Omega\rightarrow\Gamma} E(u) \quad \Rightarrow \quad \operatorname{argmin}_{u:\Omega\times\Gamma\rightarrow[0,1]} \widehat{E}(u)$$

- Possible application: Stereo matching.





# An Approach to Vectorial Total Variation based on Geometric Measure Theory

- Most real world images have multiple channels.

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# An Approach to Vectorial Total Variation based on Geometric Measure Theory

- Most real world images have multiple channels.
- How to properly generalize TV from scalar (grayscale) images to vectorial (color) images?

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# An Approach to Vectorial Total Variation based on Geometric Measure Theory

- Most real world images have multiple channels.
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- This paper considers a generalization which emerges naturally from geometric measure theory.

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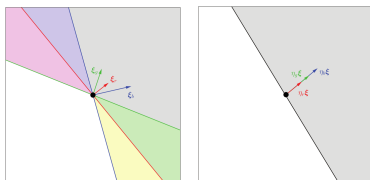


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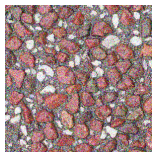
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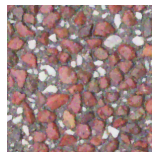
- Here color channels share a common edge direction.



Noisy Input



Naive



Proposed

# Spectral Total Variation Decomposition

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$$\partial_t u(t) = -p(t) \quad \text{s.t.} \quad p(t) \in \partial \left\| \sqrt{(\partial_x u(t))^2 + (\partial_y u(t))^2} \right\|_1$$

Generalizing what the Fourier transform is for frequencies.



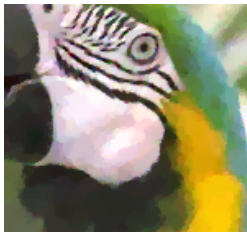
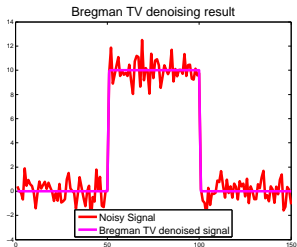
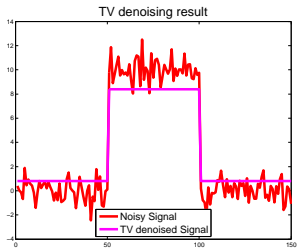
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# Bregman Iterations

$$\min_u \|Au - f\|^2 + J(u)$$

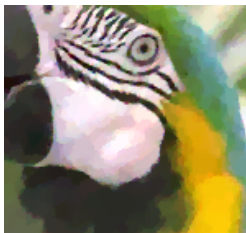
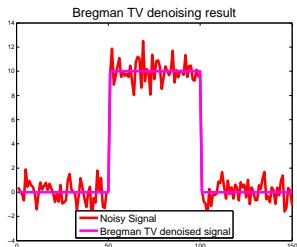
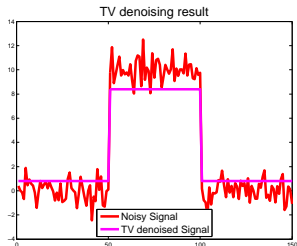
Correcting the loss of contrast of variational approaches.



# Bregman Iterations

$$\min_u \|Au - f\|^2 + J(u) - \langle p^k, u \rangle \quad \text{s.t. } p^k \in \partial J(u^k)$$

Correcting the loss of contrast of variational approaches.



## Exemplar-Based Image Inpainting

- The challenge is to remove large objects from digital images and fill in the hole in visually plausible way.
- Idea: Instead to propagating neighbouring pixel information use a non-local variational scheme and fill in using similar image patches.





# Linear Diffusion based Image Compression with iPiano

- Image compression reduces in some sense the “redundant” data in an image.

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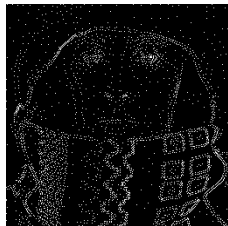
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## Linear Diffusion based Image Compression with iPiano

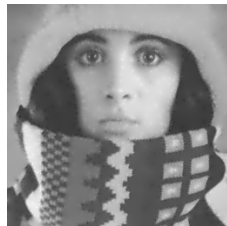
- Image compression reduces in some sense the “redundant” data in an image.
- Image inpainting can be used to restore “missing” data.
- Idea: remove pixels which are seen as redundant by the image inpainting algorithm.



Input



Compressed



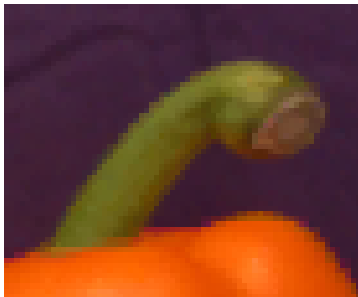
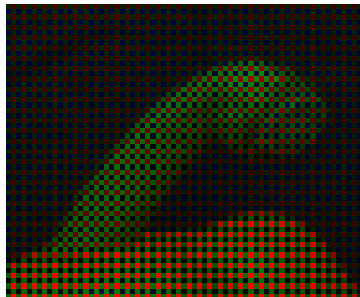
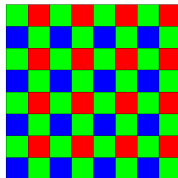
Restored

- Formulation as an *energy minimization* problem.
- Emphasis on the employed *optimization* method (iPiano).



# Image Demosaicking

- Cameras only record one color per pixel.
- Sensors have a certain pattern of colors.
- Interpolating missing colors: *Demosaicking*
- $\min_u \|P_I u - f\|^2 + J(u)$



## Dithering by Differences of Convex Functions

- Dithering aims to create the illusion of a *continuous* image given only a limited set of colors.
- Applications in printing and non-photorealistic / artistic rendering.



Input (8 bit)



Dithered (1 bit)



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- Physically motivated model based on electrostatic principles.



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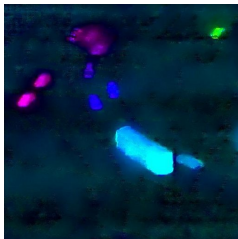
- Physically motivated model based on electrostatic principles.
- Energy minimization problem, emphasis on employed optimization method (“DC Programming”).



# Variational Optical Flow

- Given two consecutive images, one looks for a motion field which maps corresponding pixels to one another.
- The overall problem can be formulated in a variational framework via an optimization problem:

$$\min_v \int_{\Omega} |I_0(x) - I_1(x + v(x))|^2 + J(v)$$



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## Variational Super Resolution

- Given a set of low resolution images, construct a high resolution image.
- Exploit redundancy of the input frames and solve an optimization problem of the form:

$$\min_u \sum_i \|DBW_i u - f_i\|_2^2 + J(u)$$

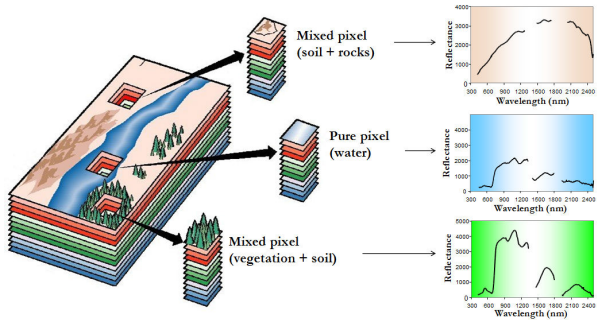
16 input images



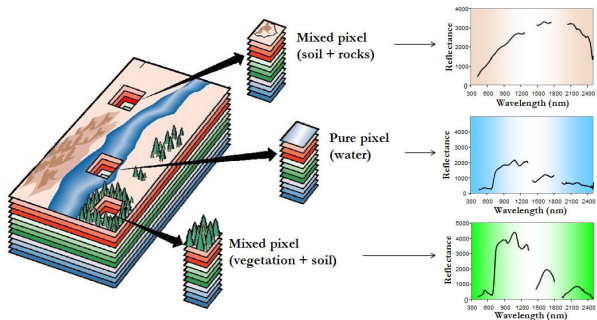
Super-resolution  $\xi = 3$



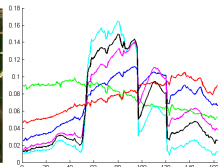
# Hyperspectral Unmixing



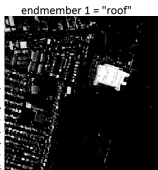
# Hyperspectral Unmixing



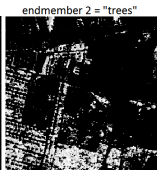
endmember 3 = "grass"



endmember 4 = "soil/dirt"



endmember 5 = "road"



endmember 6 = "different vegetation"

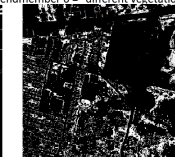


Image fusion with local and nonlocal priors:

$$\min_u \frac{1}{2} \left\| \sum_i \alpha_i u_i - f \right\|^2 + \frac{\mu}{2} \sum_i \| (\downarrow k) * u_i - g_i \|^2 + J(u)$$

**High res. gray scale  $f$  + low res. color  $g$  = high res. color  $u$**



+



=



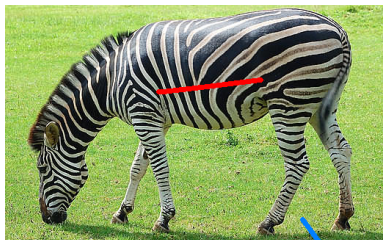
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# Globally Optimal Two Phase Segmentation

- We wish to dissect an input image in two spatially consistent segments.
- Encode the segmentation as a binary valued function  $u(x) \in \{0, 1\}$  and solve the following optimization problem:

$$\min_u J(u) + \int_{\Omega} u(x)\rho(x) dx$$



# Multilabel Segmentation

- We wish to find multiple consistent regions of an input image based on user input, color or texture.
- Formulate this combinatorial problem as a variational problem in the following way:

$$\min_u \sum_i^n \int_{\Omega} u_i(x) \rho_i(x) dx + J(u)$$



# Singular Vectors of Variational Regularizations

Disclaimer: This is a very theoretical topic!

Well known: singular value decomposition of matrices.

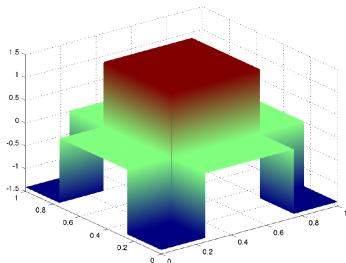
$$K^T K u = \alpha u$$

define  $\lambda = 1/\alpha$ ,  $J(u) = \frac{1}{2} \|u\|^2$ . Generalization

$$\lambda K^T K u \in \partial J(u)$$

What happens for other  $J$ , e.g.  $J(u) = TV(u)$ ?

- Is there an orthonormal basis of singular vectors?
- What properties do singular vectors have?



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End



Any questions?

These slides will be available online at  
<https://vision.in.tum.de/teaching/ws2014/vms2014>  
Password: *imageprocessing*

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