Computer Vision Group
Institut für Informatik
Technische Universität München

Weekly Exercises 8

Room: 02.09.023

Tuesday, 16.12.2015, 14:15-15:45

Submission deadline: Tuesday, 16.12.2015, 11:15 , Room 02.09.023

Higher-Order Reduction Methods (4 Points)

Exercise 1 (4 Points)**.** Consider the following pseudo-Boolean function $f : \mathbb{B}^5 \to \mathbb{R}$

$$f(x_1, \ldots, x_5) = 5x_1x_2 - 7x_1x_2x_3x_4 + 2x_1x_2x_3x_5. \tag{1}$$

a) **Reduction by Substitution.** Reduce the energy (1) to an equivalent quadratic pseudo-Boolean function by substituting products of two variables by a new variable and introducing constraints.

b) **Reduction by Minimum Selection.** Apply Ishikawa's extended minimum selection reduction method to energy (1).

Multi-Linear Polynomial Representation (4 Points)

Exercise 2 (4 Points)**.** Let $E : \mathbb{B}^n \to \mathbb{R}, n \in \mathbb{N}$, be a pseudo-boolean function. Clearly it can be written as the multi-linear polynomial

$$E(x_1, \ldots, x_n) = \sum_{\alpha \subset \{1,\ldots,n\}} a_\alpha \mathbf{x}_\alpha, \tag{2}$$

where $\mathbf{x}_\alpha = \Pi_{i\in\alpha} x_i$ and $a_\alpha \in \mathbb{R}$ are the coefficients, defining E. Now let $E_\beta \in \mathbb{R}, \beta \subset \{1, \ldots, n\}$, be defined by $E_\beta = E(x_i = 1, i \in \beta, x_j = 0, j \in \{1, \ldots, n\} \setminus \beta)$. For example let $n = 3$, then $E_{\{1,2\}} = E(1, 1, 0)$ and $E_{\{2\}} = E(0, 1, 0)$.

a) Show that the coefficients a_α are uniquely determined by the following recursive formula:

$$a_\alpha = E_\alpha - \sum_{\beta \subsetneq \alpha} a_\beta. \tag{3}$$

b) Use the recursive formula (3) to derive the coefficients a_α of the multi-linear polynomial of E, where E is given by:

x_1	x_2	x_3	$E(x_1, x_2, x_3)$
0	0	0	2
0	0	1	-13
0	1	0	4
0	1	1	5
1	0	0	-7
1	0	1	3
1	1	0	9
1	1	1	8

Programming (10 Points)

The goal of this exercise is to perform semantic multilabel segmentation of an image $I : \Omega \to \mathbb{R}^3$ using a contrast sensitive Potts regularizer. For the chosen dataset, the labels $\mathcal{L} = \{1, \ldots, 21\}$ denote various classes such as `background`, `horse`, ... The goal is to find a segmentation $x \in \mathcal{L}^n$ by minimizing the following energy:

$$E(x) = \sum_{i=1}^{n} f_i(x_i) + \sum_{i=1}^{n} \sum_{j \in \mathcal{N}(i)} f_{ij}(x_i, x_j). \tag{4}$$

The unary potentials $f_i : \mathcal{L}^n \to \mathbb{R}$ for some example images are provided as `.hdf5` files in the supplementary material `08_supp.zip` and were computed using a state-of-the-art Convolutional Neural Network[1]. The output of the network is a score for every class $s_i \in \mathbb{R}^{21}$ at each pixel $i \in \Omega$. It is stored in a pixel-first fashion, i.e. to access the cost of the l-th label at pixel (x, y) use the linear index `data[x+y*W+l*W*H]`. Compute the unary potentials as

$$f_i(x_i) = -s_i, \ \forall i \in \Omega.$$

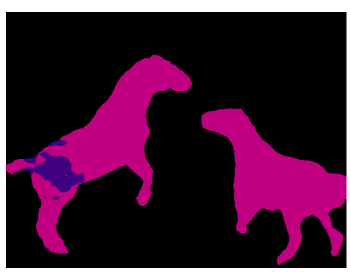
For the pairwise potentials, use the contrast sensitive Potts-model:

$$f_{ij}(x_i, x_j) = \begin{cases} 0, & \text{if } x_i = x_j, \\ g(i,j), & \text{otherwise,} \end{cases}$$

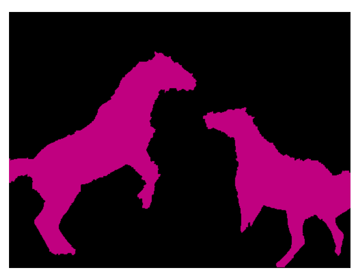
where $g(i,j) = \lambda \exp(-\gamma |I_i - I_j|)$ for some parameters $\lambda > 0$, $\gamma > 0$. Here $I_i \in \mathbb{R}^3$ denotes the color of pixel $i \in \Omega$ in the image $I : \Omega \to \mathbb{R}^3$.

Implement the model in OpenGM using `segmentation.cpp` provided in the supplementary material as a starting point. For inference, use the α-Expansion method.

Input



Unaries only



Potts

[1]Fully Convolutional Networks for Semantic Segmentation. `www.cs.berkeley.edu/~jonlong/long_shelhamer_fcn.pdf`